OPTIMIZATION OF A STAND- Alone HYBRID WIND/SOLAR
GENERATION SYSTEM FOR LIFESPAN EXTENSION

by

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Optimization of a Stand-Alone Hybrid Wind/Solar Generation System for Lifespan Extension

Thesis directed by Assistant Professor Fernando Mancilla-David and Assistant Professor Alexander Engau

ABSTRACT

The work presented in this thesis focuses on the optimization of a hybrid wind/solar system with energy storage for equipment lifespan extension. In general, electrical equipment is exposed to operational disturbances that result in temporary or permanent damage to the equipment. This study analyzes each component of the system, identifies several potential operational disturbances, and subsequently proposes a method to minimize the conditions that create or cause such effects. These conditions are minimized by solving an optimization problem that calculates the sources’ supply set points at which the system’s operational disturbances are minimized. Scenarios with different operating conditions to verify the performance of the proposed formulation along with illustrative examples in terms of case studies are also presented.
This abstract accurately represents the content of the candidate’s thesis. We recommend its publication.

Signed ____________________________
Fernando Mancilla-David

Signed ____________________________
Alexander Engau
DEDICATION

This Master’s Thesis is dedicated to my Lord and Savior Jesus Christ for His is all the power, the honor and glory. Amen.
ACKNOWLEDGMENT

I would like to sincerely thank Professor Fernando Mancilla-David for his support and teaching throughout my entire career. His teaching methodology, heavily based on conceptual foundation, combined with state-of-the-art technology applications has forged my engineering abilities and professional development.

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I would like to also dedicate this work to my brother and best friend Julian for his support and his unconditional love, support and friendship.

My mother and my father have always been there for me and given me comfort and support. Their love and support has given me the ability and courage to accomplish all my goals.
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SYMBOLS AND NOMENCLATURE

Decision variables:
\[ d_{pv} \] - Solar system DC/DC converter control signal
\[ d_{w} \] - Wind system DC/DC converter control signal

System inputs:
\[ S \] - Solar irradiation
\[ T \] - Solar panel temperature
\[ v_{DC} \] - DC bus voltage
\[ V \] - Wind speed
\[ \omega_{m} \] - Wind turbine mechanical shaft speed

Variables:
\[ f_{1}(x) \] - THD objective function
\[ f_{2}(x) \] - Battery current objective function
\[ i_{d} \] - PSMG direct current in the rotor reference
\[ i_{o,w} \] - Wind system output current
\[ i_{o,max,w} \] - Wind system maximum available output current
\[ i_{o,pv} \] - Solar system output current
\[ i_{o,max,pv} \] - Solar system maximum available output current
\[ i_{q} \] - PSMG quadrature current in the rotor reference
\[ i_{ph} \] - Photocurrent
\[ i_{rs} \] - Cell short circuit current
\[ P_{pv,ref} \] - Reference power demanded from the sun
\[ P_{pv,max} \] - Maximum solar power point
\[ P_{w,ref} \] - Reference power demanded from the wind
\[ T_{e} \] - Wind turbine electrical load torque
\[ THD_{vw}(x) \] - Total harmonic distortion
\[ T_{m} \] - Wind turbine mechanical load torque
\[ v_{s} \] - Wind generator terminal voltage
\[ W_{1} \] - Weight value associated with \( f_{1}(x) \)
\[ W_{2} \] - Weight value associated with \( f_{2}(x) \)
\[ z_{i,U} \] - Pareto set lower bound (Utopia point)
\[ z_{i,N} \] - Pareto set upper bound (Nadir point)
\[ \omega_{e} \] - Wind turbine electrical shaft speed
\[ \omega_{m,opt} \] - Maximum turbine shaft speed
Constants:

- A - p-n junction ideality factor
- A_w - Wind turbine swept Area
- C - Battery capacity
- C_b - Battery capacitance
- C_p(λ, ϑ) - Power coefficient
- e_b - Battery counter emf voltage
- E_G - Band-gap energy
- i_bmax - Battery maximum charging current
- i_rr - Reverse saturation current
- i_scr - Cell reverse saturation current
- J - Wind turbine inertia
- k - Boltzmann’s constant
- k_t - Short circuit current temperature coefficient
- L - Wind generator per phase inductance
- L_h - Stator and rotor leakage inductance
- L_d - Wind generator stator direct inductance
- L_q - Wind generator stator quadrature inductance
- L_pv - Solar system DC/DC converter output inductor
- L_w - Wind system DC/DC converter output inductor
- m_c - Number of cell in a battery
- m_p - Number of batteries in parallel
- m_s - Number of batteries in series
- n_p - Number of solar modules in parallel
- n_s - Number of solar modules in series
- P - Wind generator number of poles
- q - Charge of an electron
- r_s - Wind generator synchronous resistance
- R - Wind turbine radius
- R_i - Battery internal resistance
- T_r - Cell reference temperature
- v_gas - Battery gassing voltage
- ϑ - Blade Pitch Angle
- λ_opt - Optimum tip speed ratio
- ρ - Air density
- φ_m - Stator windings linked flux
- ξ - Allowable depth of discharge
Abbreviations:

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<td>Blade pitch angle</td>
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<tr>
<td>DNS</td>
<td>Demand not served</td>
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<tr>
<td>DOE</td>
<td>Department of energy</td>
</tr>
<tr>
<td>emf</td>
<td>Electromotive force</td>
</tr>
<tr>
<td>IncCond</td>
<td>Incremental conductance</td>
</tr>
<tr>
<td>KKT</td>
<td>KarushKuhnTucker</td>
</tr>
<tr>
<td>MTBF</td>
<td>Meantime between failure</td>
</tr>
<tr>
<td>O&amp;M</td>
<td>Operation and maintenance</td>
</tr>
<tr>
<td>P&amp;O</td>
<td>Perturb and observe</td>
</tr>
<tr>
<td>PMSG</td>
<td>Permanent magnet synchronous generator</td>
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<tr>
<td>PV</td>
<td>Photovoltaic</td>
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<tr>
<td>REEPS</td>
<td>Renewable energy electric power systems</td>
</tr>
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<td>RMS</td>
<td>Root mean square</td>
</tr>
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<td>THD</td>
<td>Total harmonic distortion</td>
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<tr>
<td>TSR</td>
<td>Tip speed ratio</td>
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1. Introduction

During the last few decades, renewable energy electric power systems (REEPS) have grown significantly. Several projects have been developed worldwide, which seek the production of clean energy. Many countries nowadays support the implementation of REEPS. For instance, China has implemented programs like the “Golden Sun Program” which consists of supporting 500 MW or more of photovoltaic power [3]. Similarly, the U.S. government awarded 500 million dollars for clean energy projects, as stated by a press release from the U.S. Department of Energy in September, 2009 [4].

In the U.S., REEPS correspond to 8% of the total energy consumption. The installed capacity as of 2009 consists of 10.6% of the total electrical generation [4]. The total generating capacity growth from 2008 to 2009 was approximately 1.5% from which wind energy represented 63.3%. Wind and solar energy represent, respectively, 9% and 1% of the U.S. renewable energy consumption according to the 2009 Annual Energy Review [5]. These statistical analyses performed by the Department of Energy clearly illustrate the magnitude of the growth corresponding to REEPS.

1.1 Hybrid Systems Review

Hybrid power systems are designed for the generation and use of electrical power from different energy sources. They are independent of a large centralized electricity grid, incorporate more than one power source or more than one fuel source for the same device, and are typically found in remote locations such as rural areas or places with low economical resources [6].
The different generating sources are meant to complement each other. Most of the time these hybrid systems are used as backup systems or operate in conjunction with the grid. The main benefit of a hybrid system is to improve reliability and efficiency. Some of the common hybrid combinations are found in the hybrid technologies matrix published by the DOE, shown in figure 1.2.

For the most part, these combinations are dictated by the geographical location and the average local weather conditions. In this study, we analyze the hybrid model studied in [7]. This model consists of a stand-alone wind/solar hybrid generation system with energy storage as shown in figure 1.1.

**Figure 1.1:** Wind/solar hybrid generation system
Figure 1.2: Energy technologies hybrid combination matrix [1]
1.2 Optimization in Power Systems

The efficient and optimal economic operation and planning of electric power generation systems have always been important in the electric power industry. The efficient use of fuel has become more relevant, primarily because most of the fuel used represents costs and irreplaceable natural resources [8]. For our study, fuel costs do not represent a concern since wind and solar energy resources are free. Energy costs are composed of fixed and variable costs. Fixed costs account for interests to loans, returns for investors, fixed operation and maintenance (O&M) charges, taxes, etc. Variable costs depend mainly on the cost of fuel and variable O&M costs [9].

Since the fuel expense is not taken into consideration, the overall cost of wind and solar power generation systems is driven by the capital costs and O&M expenses. Moreover, a major contribution to capital and O&M costs are expenses related to energy storage [10, 11]. This means that we could have economical benefits if the system is operated in such a way that the mean time between failure (MTBF) is reduced and scheduled O&M are increased.
2. System Models Review

The battery system consists of a bank of batteries connected in series to obtain the desired DC link voltage and connected in parallel to increase capacity. Each battery contains a number of cells. For this study, 6-cell batteries are used with 2.1 Volts per cell. The battery bank’s nominal voltage is 48 Volts and the nominal capacity is 40 A-hr per battery string. Other parameters are given in Appendix A.

The equivalent circuit of the battery bank system is shown in figure 2.1. Here, $e_b$ and $v_{DC}$ denote the electromotive force (emf) voltage and the terminal voltage respectively; $i_b$ is the battery bank current. The capacitance $C_b$ of the batteries is neglected for this study. The resistance $R_i$ is typically small, in the order of tenths of an Ohm.

![Equivalent circuit of a battery](image)

**Figure 2.1:** Equivalent circuit of a battery
The equation for the circuit in figure 2.1, ignoring the battery capacitance $C_b$ is

$$v_{DC} = e_b + i_b R_i,$$  \hspace{1cm} (2.1)

therefore the current following at any given time is

$$i_b = \frac{v_{DC} - e_b}{R_i}. \hspace{1cm} (2.2)$$

The direction of the current of the battery bank $i_b$ is dictated by the terminal voltage. When the voltage of the battery $e_b$ is equal to the charging source voltage $v_{DC}$, no current will flow. When the voltage of the battery is less than that of the charging source, current will flow into the battery and charge it, but if the battery voltage is higher, current will flow out of the battery and discharge it [12].

2.1 Wind Subsystem

The wind subsystem is formed by a permanent magnet synchronous generator (PMSG), a wind turbine, a passive rectifier and a buck DC/DC converter as shown in figure 2.2.

![Wind Subsystem Block Diagram](image)

**Figure 2.2:** Wind subsystem block diagram
The mechanical power extracted from the wind turbine is directly related to the wind velocity. This means that the wind turbine will only produce as much power as wind availability allows. This relationship is defined as

$$ P_w = \frac{1}{2} C_p(\lambda, \vartheta) \rho A V^3, $$

where $C_p(\lambda, \vartheta)$ is the rotor efficiency as a function of the tip speed ratio $\lambda$ (TSR) and the blade pitch angle $\vartheta$ (BPA). By $\rho$ we denote the air density correspondent to the altitude of the location, $A = \pi r^2$ is the cross-sectional area of the turbine with blade radius $r$, and $V$ is the wind velocity. The TSR is defined as

$$ \lambda = \frac{r \omega_m}{V}, $$

where $\omega_m$ is the angular shaft speed.

![Figure 2.3: Analytical approximation of $C_p - \lambda$ characteristics (blade pitch angle $\vartheta$ as parameter [2])]  

The rotor efficiency $C_p - \lambda$ curves from figure 2.3 may be approximated in closed form by non-linear functions. Following the turbine characteristics of [2], a model can be derived in the form:

$$ C_p(\lambda, \vartheta) = C_1 \left[ C_2 \frac{1}{\lambda_{\text{ref}}} - C_3 \vartheta - C_4 \vartheta^y - C_5 \right] e^{-C_6 \frac{1}{\lambda_{\text{ref}}}}, $$

$$ (2.5) $$
where
\[
\frac{1}{\lambda_{\text{ref}}} = \frac{1}{\lambda + 0.08\theta} - \frac{0.035}{\theta^3 + 1},
\] (2.6)
and
\[
C_1 = \frac{1}{2}, \quad C_2 = 116, \quad C_3 = 0.4
\]
\[
C_4 = 0, \quad C_5 = 5, \quad C_6 = 21
\] (2.7)
are coefficients that are obtained from the model developed in [2].

For this study the turbine’s BPA is constant and equal to zero and therefore
\[
C_p(\lambda) = \frac{1}{2} \left[ 116 \left( \frac{1}{\lambda} - 0.035 \right) - 5 \right] e^{-21(\frac{1}{\lambda} - 0.035)},
\] (2.8)

2.1.1 Wind Subsystem Operation

The mathematical definition of the PMSG’s dynamic performance in a rotor reference frame based on the model represented in figure 2.4 is
\[
\dot{i}_q = -\frac{r_s}{L} i_q - \frac{P}{2} \omega_m \phi_m + \frac{P}{2} \frac{\omega_m \phi_m}{L} \frac{\pi v_{\text{DC}} i_q}{3\sqrt{3}L \sqrt{i_d^2 + i_q^2}} \frac{1}{d_w},
\] (2.9)
\[
\dot{i}_d = -\frac{r_s}{L} i_d + \frac{P}{2} \omega_m \phi_m - \frac{\pi v_{\text{DC}} i_d}{3\sqrt{3}L \sqrt{i_d^2 + i_q^2}} \frac{1}{d_w},
\] (2.10)
\[
\dot{\omega}_e = \frac{P}{2J} \left( -\frac{3}{2} \frac{P}{2} \phi_m i_q + \frac{1}{2} \frac{C_p(\lambda) \rho A V^3}{\omega_m \frac{\omega_m}{T_t}} \right),
\] (2.11)
where \(L\) represents the stator per phase inductances \(L_d = L_q = L\); \(r_s\) is the stator per phase resistance; \(i_d\) and \(i_q\), are respectively, the direct and quadrature current in the rotor reference frame; \(\phi_m\) is the flux linked by the stator windings; \(T_t\) is the wind turbine torque; \(J\) is the inertia of the rotating parts; \(P\) is the number of poles of the PMSG; and \(d_w\) is the control signal (duty ratio) of the wind system DC/DC converter [7].
For steady state operation $i_q = 0$, $i_d = 0$, and $\omega_e = 0$ which result in

\begin{align}
- \frac{r_s}{L} i_q - \frac{P}{2} \omega_m i_d + \frac{P}{2} \frac{\omega_m \phi_m}{L} - \frac{\pi v_{DC} i_q}{3\sqrt{3}L\sqrt{i_d^2 + i_q^2}} \frac{1}{d_w} &= 0, \quad (2.12) \\
- \frac{r_s}{L} i_d + \frac{P}{2} \omega_m i_q - \frac{\pi v_{DC} i_d}{3\sqrt{3}L\sqrt{i_d^2 + i_q^2}} \frac{1}{d_w} &= 0, \quad (2.13) \\
- \frac{3}{2} \frac{P}{2} \frac{\phi_m i_q}{2} + \frac{1}{\omega_m} \frac{C_p(\lambda) \rho AV^3}{\omega_m} &= 0. \quad (2.14)
\end{align}

The expression for the output current of the wind subsystem DC/DC converter is:

$$i_{o,w} = \left( \frac{\pi}{2\sqrt{3}} \sqrt{i_d^2 + i_q^2} \right) \frac{1}{d_w}. \quad (2.15)$$

In order to calculate the output current, $d_w$, $i_d$, $i_q$, $V$ and $\omega_e$ must be known. The control signal $d_w$ is adjusted by the control system depending on the rotor shaft speed $\omega_m$. If the wind velocity $V$ is known, then $\omega_m$ may be calculated using the steady state set of equations (2.12) to (2.14). However, if $\omega_m$ is measured, the wind velocity measurement is not necessary since $i_d$ and $i_q$...
may be calculated using equations (2.12) and (2.13) only.

The power demanded to the wind subsystem is dictated by the following reference:

\[ P_{w_{\text{ref}}} = v_{DC}(i_L + i_b - i_{o,pv}), \]  

(2.16)

where \( i_L \) is the load demand current, and \( i_{o,pv} \) is the current contribution from the solar system.

In order to determine if the wind system is able to sustain the power demand \( P_{w_{\text{ref}}} \) we need to know the maximum power that can be extracted from the generator. Due to the concavity of the \( C_p - \lambda \) curves, it is possible to calculate the value of the TSR (\( \lambda_{\text{opt}} \)) in which the rotor efficiency \( C_p \) is at its maximum. Figure 2.5 shows the optimum TSR for a BPA of 0° where \( \lambda_{\text{opt}} = 7.954 \).

\[ \lambda_{\text{opt}} \] at a blade pitch angle \( \vartheta = 0^\circ \)

**Figure 2.5:** \( \lambda_{\text{opt}} \) at a blade pitch angle \( \vartheta = 0^\circ \)
Given that $\lambda_{\text{opt}}$ is known, the maximum power that can be extracted from the wind generator is

$$P_{\text{wmax}} = \left( \frac{C_p(\lambda_{\text{opt}})\rho AR^3}{2\lambda_{\text{opt}}^3 \kappa_{\text{opt}}} \right) \omega_m^3 - \frac{3}{2} \left( i_q^2 + i_d^2 \right) r_s^3 P_{\text{wloss}}. \quad (2.17)$$

With equation (2.17) we may obtain an expression for the optimum shaft speed where the power extraction is at its maximum [7]. Solving equation (2.17) for $\omega_m$ we obtain

$$\omega_{m_{\text{opt}}} = \sqrt[3]{\frac{P_{\text{wref}} + \frac{3}{2} \left( i_q^2 + i_d^2 \right) r_s}{C_p(\lambda_{\text{opt}})\rho AR^3 \kappa_{\text{opt}}^3}}. \quad (2.18)$$

This relationship allows us to compare the measured shaft speed with the optimum shaft speed and determine the value of the maximum available power at any given time.

In order to control the amount of power extracted from the wind turbine, the machine torque must be adjusted. Adjusting the machine’s terminal voltage magnitude $|v_s|$ will adjust the electrical load torque using

$$T_e = \frac{3P v_s \phi_m}{4\omega_e L_s} \sqrt{1 - \left( \frac{v_s}{\omega_e \phi_m} \right)^2}, \quad (2.19)$$

where the stator resistance $r_s$ is neglected; $L_s$ is the stator and rotor leakage inductance; and $\omega_e = \frac{P}{2\omega_m}$ is the electrical shaft speed. This can be achieved by adjusting the control signal $d_w$ of the DC/DC converter from

$$|v_s| = \frac{1}{d_w} \left( \frac{\pi v_{\text{DC}}}{3\sqrt{3}} \right). \quad (2.20)$$
2.2 Solar Subsystem

The Solar subsystem consists of an array of solar panels and a DC/DC buck converter as illustrated in figure 2.6.

![Solar subsystem block diagram](image)

**Figure 2.6:** Solar subsystem block diagram

The solar panels are formed by photovoltaic cells, which are $p-n$ semiconductor junctions that convert solar energy into electricity. Figure 2.7 illustrates the equivalent circuit of a photovoltaic cell.

![Solar cell equivalent circuit](image)

**Figure 2.7:** Solar cell equivalent circuit
The current extracted from the photovoltaic array is defined as

\[ i_{pv} = n_p i_{ph} - n_p i_{rs} \left( e^{q \left( \frac{v_{pv} + i_{pv} R_s}{n_s A K T} \right)} - 1 \right), \]  

(2.21)

where \( n_s \) is the number of cells connected in series; \( n_p \) accounts for the modules connected in parallel; \( v_{pv} \) is the output voltage at the PV terminals; \( q \) is the charge of an electron; \( R_s \) is the intrinsic cell series resistance (for simplicity the series and shunt resistance, \( R_s \) and \( R_{sh} \) are neglected); \( A \) is the cell deviation from the ideal \( p - n \) junction; \( K \) is the Boltzmann’s constant; \( T \) is the cell temperature.

Moreover, \( i_{ph} \) and \( i_{rs} \) are defined as

\[ i_{ph} = [i_{sc} + k_t (T - T_r)] \frac{S}{100}, \]  

(2.22)

\[ i_{rs} = i_{rr} \left( \frac{T}{T_r} \right)^3 e^{\frac{a E_G}{K A} \left[ \frac{1}{T_r} - \frac{1}{T} \right]}, \]  

(2.23)

where \( k_t \) is the short circuit current temperature coefficient; \( T_r \) is the cell reference temperature; \( i_{sc} \) is the short circuit current at reference temperature and radiation; \( S \) is the solar radiation in \([\text{mW/cm}^2]\); \( i_{rr} \) is the reverse saturation current; and \( E_G \) is the band-gap energy of the semiconductor used.

### 2.2.1 Solar Subsystem Operation

The dynamic mathematical model of the solar system is:

\[ \dot{i}_{o,pv} = -\frac{v_{DC}}{L_{pv}} + \frac{v_{pv}}{L_{pv}} \]  

(2.24)

\[ \dot{v}_{pv} = \frac{i_{pv}}{C_{pv}} - \frac{i_{o,pv}}{C_{pv}}, \]  

(2.25)

where \( L_{pv} \) and \( C_{pv} \) are the solar system DC/DC converter pole inductance and throw capacitance respectively. For steady state operation \( \dot{i}_{o,pv} = 0 \) and \( \dot{v}_{pv} = 0 \).
Thus, the expression for the output current of the solar subsystem is:

\[ i_{o,pv} = i_{pv} \frac{1}{d_w}. \]  

(2.26)

In order to calculate amount of power injected by the photovoltaic array, the solar irradiation \( S \) and the cell temperature \( T \) must be measured. The current
available from the solar panel is given by one of the I-V curves in figure 2.8 which depends on these parameters.

The power demanded to the solar subsystem is dictated by the following reference:

\[ P_{p\text{ref}} = v_{DC}(i_L + i_b - i_{o,w}). \] (2.27)

In order to determine if the solar system is able to sustain the power demanded from equation (2.27) we need to know the maximum power that can be extracted from the photovoltaic array. The power demand is driven by the load demand and the battery charging current demand as illustrated in equation (2.27).

The maximum solar power available is calculated using a technique called IncCond (Incremental Conductance) developed in [13]. This technique is an improved algorithm of the conventional Perturb and Observe (P&O) method. This technique is used to constantly track the maximum power available from the solar system at any time. The photovoltaic array output power, ignoring \( R_s \) and \( R_{sh} \) may be calculated using equation (2.21) as follows:

\[ P_{pv} = v_{pv}n_p i_{ph} - v_{pv}n_p i_{rs} \left( e^{\left(\frac{v_{pv}s}{n_p A k T}\right)} - 1 \right). \] (2.28)

Given the characteristics of the I-V and P-V curves of the solar panel in figure 2.8, the maximum power that can be extracted from the solar panels is obtained from

\[ \frac{\delta P_{pv}}{\delta v_{pv}} = \frac{\delta (i_{pv}v_{pv})}{\delta v_{pv}} = \frac{\delta i_{pv}}{\delta v_{pv}} v_{pv} + i_{pv} = 0. \] (2.29)

resulting in

\[ P_{pv\text{max}} = -\frac{\delta i_{pv}}{\delta v_{pv}} v_{pv}^2 = \frac{n_p q}{n_s k T A} i_{rs} v_{pv}^2 e^{\left(\frac{v_{pv} s}{n_p A k T}\right)}. \] (2.30)
Figure 2.9: PV Curves maximum values at 25°C

Subsequently, the set of curves from figure 2.9, obtained using equation (2.30), show the maximum available power at any given time independent of the solar irradiation $S$.

Once the maximum power for both systems is known, we are able to simulate modes of operation as outlined in chapter 4. The following chapter prepares this discussion by analyzing the individual operation of each subsystem described in this chapter.
3. **System Operational Contingencies Analysis**

Power systems economics play a very important role in the engineering and planning portion of the design. As mentioned before, power systems take into consideration two major economical factors: fixed and variable costs. Fixed costs account for interests on loans, returns to investors, fixed operation and maintenance (O&M) charges, taxes, etc. Variable costs depend mainly on the cost of fuel and actual O&M of the plant [9].

Power systems optimization aims at operating the system at the lowest possible cost without violating established constraints. For wind and solar systems fuel expenses do not exist and therefore the economical analysis takes into account other variable costs such as O&M. For this reason, it is important to analyze each systems individually to identify the operational disturbances that could cause malfunctions and thereby reduce the mean time between failure (MTBF).

### 3.1 Battery Potential Hazards

The proper operation of the batteries is critical to the performance of the system. Batteries represent a significant part of the capital/O&M costs in the entire system. In a study composed of a PV systems with energy storage, batteries accounted for more than 40% of the capital cost [11]. For this reason it is important to address the scenarios where the lifespan of the batteries could be compromised.

Some of the scenarios that could potentially decrease the lifespan of batteries are the following [14]:
• discharging the batteries below the allowable percentage
• overcharging the batteries
• undercharging
• extreme operating temperatures

Most of these scenarios are able to be reduced by properly managing the state of charge of the batteries.

### 3.1.1 Batteries Discharging Mode

For the extent of this study, the DC bus voltage is the measuring point of the battery voltage. This is illustrated in the battery equivalent circuit from figure 2.1. When the batteries are committed, the following criteria must be taken into consideration:

\[ i_b = \begin{cases} 
    i_L - i_{o_{pv}} - i_{o_w} & \text{if } v_{DC} \geq \xi e_b, \\
    0 & \text{if } v_{DC} < \xi e_b.
\end{cases} \quad (3.1) \]

Here \( v_{DC} \) is the voltage at the DC link, \( \xi \) is a constant that represents the allowable depth of discharge (typically 80%-90% of the nominal voltage [15]), and \( i_b \) is the battery discharge current. For this study the allowable depth of discharge \( \xi \) is set to 85%.

When the batteries are being discharged, the current demanded by the load will dictate the discharge time.

### 3.1.2 Batteries Charging Mode

Similar to the discharge state of the batteries, the charging state of the batteries could become a life-reducing factor for the batteries if these become overcharged. The batteries specifications dictate the maximum amount of current...
that can be injected into the batteries. This current is $i_{b_{\text{max}}}$ and it is typically dictated by the manufacturer.

![Battery charge characteristic curve](image)

**Figure 3.1:** Battery charge characteristic curve

From chapter 1 we know that the batteries charging current depends on the batteries’ terminal voltage $v_{DC}$ following equation (2.2). During the early charge stage, when the emf voltage $e_b$ is very low, the batteries require higher currents to recover from the discharge event. The charging current is limited to the maximum allowable current $i_{b_{\text{max}}}$ during this period of bulk charging current. Once 95% of the battery voltage is recovered, the batteries will continue to be charged with the recommended trickle charge current. This method allows to maintain the batteries fully charged at a constant current without overcharging them. The trickle charge current is generally dictated by the manufacturer in
the batteries’ specifications. If a recommended trickle charge current is not provided, a conservative value of \( \frac{C}{100} \) is used, where \( C \) is the capacity of the batteries [16]. For example, if the batteries were rated as 100 Ahr, the trickle charge current would be 1A. Figure 3.1 illustrates the charge characteristic of the batteries.

With the previous information we may now determine a suitable charging function for the batteries:

\[
i_{b_{\text{spec}}} = \begin{cases} 
  i_{b_{\text{max}}} & \text{if } 0 \leq v_{DC} \leq e_b - R_i \ i_{b_{\text{max}}}, \\
  \frac{v_{DC} - e_b}{R_i} & \text{if } e_b - R_i \ i_{b_{\text{max}}} \leq v_{DC} \leq 95\% \ v_{DC}, \\
  \frac{C}{100} & \text{if } 95\% \ v_{DC} < v_{DC} < v_{gas}, \\
  0 & \text{if } v_{gas} \leq v_{DC},
\end{cases}
\]  

(3.2)

where \( v_{gas} \) is the gassing voltage. This is the voltage at which a battery will begin to produce hydrogen and oxygen, which reduces the amount of water in the battery [16]. This phenomenon causes the lifespan of the battery to be shortened.

### 3.2 Wind Subsystem Potential Hazards

Wind turbines are currently designed to last around 20 years [17]. This is achieved by diligent maintenance and operation of the turbine. The most common damaging factors of wind turbines are stresses in the drive train of the generator and overheating.

Stresses on the shaft of the generator are directly related to the torques that are subject to fluctuation due to periodic and aperiodic processes such as [2]

- changes in wind speed;
• tower shadow or tower-occasioned upwind overpressure;
• blade asymmetry;
• blade bending and skewing;
• tower oscillation.

From an electrical point of view, we only consider the changes in wind speed as a threatening factor from the list above since all other factors are mechanical/structural-related. The maximum stress occurs at the surface of the shaft when the radial position is at rest. This means that, in order to avoid stress that could potentially reduce the lifespan of the wind turbine, it is desired to maintain any speed greater than zero.

Another perturbation that reduces the life of the wind turbine is heat. This phenomenon occurs on any PMSG due to the current flow through resistive elements such as the armature windings. The armature winding losses, the core losses, and the stray losses are frequency-dependant and are directly related with the output current of the turbine. If this current is distorted with harmonics produced by the passive rectifier, these will create more losses in the system. An illustration of the relationship between harmonic currents $I_a$ and the turbine losses (heat generated in the turbine) is given by

$$P_{\text{unloss}} = \sum_{n=1}^{\infty} P_{\text{anloss}} = m_1 \sum_{n=1}^{\infty} I_{an}^2 R_{1in} \approx m_1 R_{1dc} \sum_{n=1}^{\infty} I_{an}^2 k_1 R_n$$  \hspace{1cm} (3.3)$$

where $n = 5, 7, 11, \ldots$, which represent the odd harmonics that are not multiples of three. This expression is in terms of the frequency-dependent losses
of a generator with a passive rectifier attached downstream [18] similar to this study’s application.

Furthermore, if harmonics produced by the wind generator are not controlled or suppressed, they could create damage in other components of the systems such as the batteries, the solar panel, or even the load. Additionally, enhancing the power quality in this manner would also increase efficiency.

3.3 Solar Subsystem Potential Hazards

The power output from a PV array is directly related to the solar irradiation and the cell temperature. The PV system will output a DC current that is essentially a current source with a few non-idealities. This means that this current will be free of current harmonics for the most part. Additionally, the output of the PV module is managed by the DC/DC converter that links the PV array to the system.

There are potential events that will create a malfunction in the solar panel such as hot spot, thermal cycling, moisture, or mechanical loads. These conditions are not part of the system’s operation and thus nothing that can be done to prevent these threats by means of operation. There has been research regarding hot-spot analysis and prevention [19]; however, so far a solution has not been established prevent such phenomenon from an operational perspective. Alternatively, hot spot heating and the other mentioned threats may be prevented from a fabrication perspective.
4. System Modes of Operation

The system’s modes of operation vary over time with respect to the variations in the load demand. The system is analyzed in steady state, considering the present value of the load at any given time not taking into consideration previous or future load demands.

Because the load is repeatedly changing and the wind and solar energy sources change as well, the nature of the system is to cycle. These cycles have to do with nature’s day and night cycles: during the day, the solar energy may be the predominant energy source and, similarly, the wind system may be predominant during the night time. Furthermore, the battery will be in either state of charge or disconnected. For this reason, the modes of operation may be represented by a number that would represent the ON/OFF state of each energy source.

For instance, “1” would represent the “ON” state when the unit is committed; “0” would represent the “OFF” state when the unit is uncommitted; and using a special notation for the battery, “−1” would represent its charging state. Table 4.1 contains the proposed modes of operation using the format mentioned above.

4.1 Operation Mode 1

This mode of operation corresponds to the unlikely situation in which the wind and solar sources are off and the batteries are responsible for handling the
Table 4.1: Modes of Operation

<table>
<thead>
<tr>
<th>Modes of operation</th>
<th>Unit Combination $P_w$ $P_{pv}$ $P_b$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Mode 1</td>
<td>0 0 1</td>
</tr>
<tr>
<td>Mode 2</td>
<td>0 1 -1</td>
</tr>
<tr>
<td>Mode 3</td>
<td>0 1 1</td>
</tr>
<tr>
<td>Mode 4</td>
<td>1 0 -1</td>
</tr>
<tr>
<td>Mode 5</td>
<td>1 0 1</td>
</tr>
<tr>
<td>Mode 6</td>
<td>1 1 -1</td>
</tr>
<tr>
<td>Mode 7</td>
<td>1 1 1</td>
</tr>
<tr>
<td>Mode 8</td>
<td>0 0 0</td>
</tr>
</tbody>
</table>

entire load. This mode of operation is best described as follows:

$$\begin{align*}
\text{if } \left\{ \begin{array}{l}
\omega_m = 0 \\
\omega_{pv} = 0
\end{array} \right. \Rightarrow \text{Mode 1} : & \\
\text{Wind Uncommitted, } P_w = 0 & \\
\text{Solar Uncommitted, } P_{pv} = 0 & \\
\text{Battery Committed, } P_b = v_{DC} i_b &
\end{align*}$$

(4.1)

In this case the batteries are susceptible to fast discharge if the energy is not managed properly using the procedures described in chapter 2.

If this mode remains for a long time, the batteries must be disconnected for their protection and the power will not be restored until the wind and solar sources become available and the batteries are completely recharged.

Once the monitoring equipment detects voltage at the DC link bus, the battery bank is immediately re-connected to be charged again. When the batteries voltage raise above $\xi v_{DC}$ the load may be restored using an appropriate load.
4.2 Operation Mode 2

This mode of operation occurs when the solar panel is responsible of supplying the load, including the load of the batteries when they are being charged. The wind turbine is unavailable during this mode. This situation may be encountered in weather conditions with high solar intensity and a low wind speed profile or if the wind turbine is disconnected for maintenance. Depending on how the system is sized and the load shedded, this system may or may not be functional.

This mode of operation is illustrated by the following definitions:

\[
\begin{align*}
\text{if } & \left\{ \begin{array}{l}
\omega_m = 0 \\
P_{pv,max} \geq P_{pv,ref}
\end{array} \right\} \Rightarrow \text{Mode 2 : } \\
& \begin{cases}
\text{Wind Uncommitted, } P_w = 0 \\
\text{Solar Committed, } P_{pv} = v_{DC}(i_L + i_b) \\
\text{Battery Charging, } P_b = v_{DC} i_b
\end{cases}
\end{align*}
\]

4.3 Operation Mode 3

This mode of operation will be encountered if mode 2 is in operation, and the load increases above \( P_{pv} \), so that the battery may function as a regulator for the power that the solar system cannot support. Since the batteries are now committed, the batteries must be protected against deep discharge as it is done in mode 1. This is illustrated as follows:

\[
\begin{align*}
\text{if } & \left\{ \begin{array}{l}
\omega_m = 0 \\
P_{pv,max} < P_{pv,ref}
\end{array} \right\} \Rightarrow \text{Mode 3 : } \\
& \begin{cases}
\text{Wind Uncommitted, } P_w = 0 \\
\text{Solar Committed (MPPT), } P_{pv} = -\frac{\delta P_{pv}}{\delta v_{pv}} v_{pv}^2 \\
\text{Battery Committed, } P_b = v_{DC} i_b
\end{cases}
\end{align*}
\]
4.4 Operation Mode 4

Mode 4 is similar to mode 2. In this case the load is supplied by the wind turbine. The batteries are part of the total load in this scenario since they are being charged. The solar system is unavailable. This occurs at night time and whenever the solar system is disconnected for maintenance. This scenario, as in mode 2, may not be functional if the wind system is undersized or if the load is not shedded properly.

\[
\begin{align*}
\text{if} \left\{ \begin{array}{l}
\omega_m \geq \omega_{m,\text{opt}} \\
\quad P_{pv} = 0
\end{array} \right. \Rightarrow \text{Mode 4} : \\
\begin{cases}
\text{Wind Committed, } P_w = v_{DC} (i_L + i_b) \\
\text{Solar Uncommitted, } P_{pv} = 0 \\
\text{Battery Charging, } P_b = v_{DC} i_b
\end{cases}
\end{align*}
\]

4.5 Operation Mode 5

Mode 5 is an analogous representation of mode 3, where the load is being supplied by the wind system and the batteries. This occurs if the load increases above \( P_w \) while the system is in mode 4. Similar to mode 3, the batteries function to compensate for the demand not served (DNS) by the wind turbine. The batteries must be protected from being discharged excessively.

\[
\begin{align*}
\text{if} \left\{ \begin{array}{l}
\omega_m < \omega_{m,\text{opt}} \\
\quad P_{pv} = 0
\end{array} \right. \Rightarrow \text{Mode 5} : \\
\begin{cases}
\text{Wind Committed (MWPG), } P_w = K_{opt} \omega_m^3 - P_{\text{loss}} \\
\text{Solar Uncommitted, } P_{pv} = 0 \\
\text{Battery Committed, } P_b = v_{DC} i_b
\end{cases}
\end{align*}
\]

4.6 Operation Mode 6

During this mode of operation the wind and solar systems work in conjunction to supply the load \( P_b + P_L \). This mode of operation is the most desirable
because there is more than one source available. Combining these sources enables the opportunity to cover the demand with enough flexibility to apply optimization techniques.

\[
\begin{align*}
\text{if} \left\{ \begin{array}{c}
\omega_m \geq \omega_{m\text{opt}} \\
P_{pv\text{max}} \geq P_{pv\text{ref}}
\end{array} \right\} & \Rightarrow \text{Mode 6 : } \\
\text{Wind Committed, } P_w &= v_{DC}(i_L + i_b - i_{o\text{,pv}}) \\
\text{Solar Committed, } P_{pv} &= v_{DC}(i_L + i_b - i_{o\text{,w}}) \\
\text{Battery Charging, } P_b &= v_{DC} i_b
\end{align*}
\]

4.7 Operation Mode 7

This is the mode of operation where all of the sources are committed. This will occur if the load increases above \( P_{pv} + P_w \) while mode 6 is in operation. The batteries will compensate for DNS during this mode. If the battery drops to a value below the allowable depth of discharge then the entire system would shutdown to protect the batteries.

\[
\begin{align*}
\text{if} \left\{ \begin{array}{c}
\omega_m < \omega_{m\text{opt}} \\
P_{pv\text{max}} < P_{pv\text{ref}}
\end{array} \right\} & \Rightarrow \text{Mode 7 : } \\
\text{Wind Committed, } P_w &= K_{opt}\omega^3_m - P_{w\text{loss}} \\
\text{Solar Committed, } P_{pv} &= -\frac{\delta_{pv}}{\delta_{pv}} v_{pv}^2 \\
\text{Battery Committed, } P_b &= v_{DC} i_b
\end{align*}
\]

4.8 Operation Mode 8

This mode is encountered when the system is not in operation. This mode of operation illustrates an initial condition where all components are off. This could also occur after a forced shutdown occurs. For instance, this mode would be committed if the batteries voltage drops below the allowable depth of discharge \( \xi \) in modes 3, 5, and 7.
5. Problem Formulation

The base for the formulation of the problem is operation mode 6 since it provides the most flexibility in terms of the power distribution to the load. During this scenario, as mentioned before, both sources (wind and solar) are committed to fulfill the load. In this case, the power extraction from both sources depends on the load demand. The supply of the load may be achieved with an infinite number of combinations of energy drawn by the two sources; however, some combinations may not be optimal.

In order to create a mathematical formulation, it is necessary to understand the system’s behavior in steady state. It is also important to know the reaction of the different components of the system to certain pollutants such as the harmonics injected into the system or surges as explained in chapter 3. These harmonics are created from the power electronics attached to each source and the load.

As described in chapter 3, the wind turbine is affected by the constant additional heat produced by the current harmonics. This heat creates damage in the components of the generator in the long run. Minimizing these harmonics will also minimize the damage to the wind turbine. Additional benefits are obtained from minimization of the harmonics such as reducing the losses from the generator and enhancing efficiency. It is not in the scope of this project to analyze the efficiency of the generator, but it is important to mention the achieved benefits.
The wind turbine is directly connected to a passive three phase rectifier. The output of this converter is uncontrolled, and therefore the analysis is done downstream of the rectifier. The DC current from the rectifier is then controlled using a buck DC/DC converter.

The output current of the wind subsystem is a DC current previously defined in equation (2.26). This current may contain current harmonics generated by the DC/DC converter and the wind generator resulting in produced heat at the generator core and stator windings. For this reason it is important to quantify the amount of current harmonics in the output current waveform.

5.1 Total Harmonic Distortion Index

Various harmonic components of a periodic waveform of the form \( f(t) \) with a period \( \pi \) may be readily conducted using a Fourier analysis of the waveform based on time domain integration [20]. This is because a general non-sinusoidal waveform \( f(t) \) repeating with an angular frequency \( \omega_{sw} \) can be expressed as [21]

\[
f(t) = F_0 + \sum_{n=1}^{\infty} f_n(t) = \frac{1}{2} a_0 + \sum_{n=1}^{\infty} \left\{ X_{a(n)} \cos(n\omega_{sw}t) + X_{b(n)} \sin(n\omega_{sw}t) \right\},
\]

where \( F_0 = \frac{1}{2} a_0 \) is the average value defined as

\[
F_0 = \frac{1}{\pi} \int_0^{2\pi} f(t) \, dt,
\]

and \( X_{a(n)} \) and \( X_{b(n)} \) are respectively, the odd and even components of the harmonic components:

\[
X_{a(n)} = \frac{1}{\pi} \int_0^{2\pi} f(t) \cos(n\omega_{sw}t) \, dt \quad n = 0, 1, \ldots, \infty
\]

\[
X_{b(n)} = \frac{1}{\pi} \int_0^{2\pi} f(t) \sin(n\omega_{sw}t) \, dt \quad n = 0, 1, \ldots, \infty.
\]

For our study the average value from equation (5.2) is non-zero because it the waveform is non-symmetrical. The expressions of the odd (cosine) and even
(sine) components of the $n$th harmonic of the wind system output current $i_{o,w}$ are

$$i_{\cos} = \frac{2}{T_{sw}} \int_0^{d_{sw} T_{sw}} i_{o,w} \cos(n \omega_{sw} t) \, dt = \frac{i_{o,w} \sin(2\pi nd_w)}{\pi n}$$

$$i_{\sin} = \frac{2}{T_{sw}} \int_0^{d_{sw} T_{sw}} i_{o,w} \sin(n \omega_{sw} t) \, dt = i_{o,w} \left( \frac{1}{\pi n} - \frac{\cos(2\pi nd_w)}{\pi n} \right).$$

(5.4)

where $T_{sw}$ is the switching period and $\omega_{sw}$ is the angular frequency defined as

$$\omega_{sw} = 2\pi F_{sw}$$

(5.5)

where

$$F_{sw} = \frac{1}{T_{sw}}$$

(5.6)

The complex phasor form of the $n$th harmonic of $i_{o,w}$, for all positive integer values of $n$ is

$$i_{w,\text{vect}} = i_{\cos} - j i_{\sin}. \quad (5.7)$$

Due to symmetry, if the function is odd, the odd component of the $n$th harmonic will be zero for all values of $n$. If the function is even, it would similarly result in zeros for the even component of the $n$th harmonic for all values of $n$.

Moreover, the magnitude of the complex phasor from equation (5.7) is

$$|i_{w,\text{vect}}| = \sqrt{i_{\cos}^2 + i_{\sin}^2} = \frac{2i_{o,w}}{\pi n} \sin(\pi nd_w). \quad (5.8)$$

The amount of distortion in the voltage or current waveforms is quantified by means of an index called the Total Harmonic Distortion (THD) [21]. This THD index is defined as the ratio of the RMS of the harmonic content to the RMS value of the fundamental quantity, expressed as a percent of the fundamental [22]

$$THD_{i_w}(x) = \sqrt{\sum_{n=1}^{\infty} \left| \frac{i_{\text{vect}}}{|i_{\text{vect},0}|^2} \right|^2}.$$  

(5.9)

30
After substituting the correspondent variables and constants, the expression becomes
\[
THD_{tw}(x) = \sqrt{\sum_{n=1}^{\infty} \left[ \frac{1}{\pi n} \sin(\pi nd_{w}) \right]^2 \frac{d_{w}}{d_{w}^2}}.
\] (5.10)

5.2 Mathematical Optimization Model

One of the objectives of the optimization problem is to minimize the amount of harmonics produced by the wind generator (\(THD_{tw}\)). Now that these harmonics have been quantified and represented by a mathematical expression, it is possible to formulate the optimization problem.

It should be noted that the expression of the THD in equation (5.10) as well as the equality constraints and some of the inequality constraints are non-linear.

Furthermore, the battery system requires the charging and discharge current to match the manufacturer’s charging and discharge recommended parameters as closely as possible. This means that ideally \(i_b = i_{b\text{spec}}\). In order to ensure such expression to be fulfilled, the absolute or squared residual could be minimized and become a second objective function.
The resulting optimization model is given as

\[
\begin{align*}
\min_x \quad & f_1(x) = THD_{\omega}^2 = \sum_{n=1}^{\infty} \left( \frac{1}{\pi n} \sin(\pi nd_w) \right)^2 \\
\min_x \quad & f_2(x) = [b_{bpec} - b_b]^2 = [b_{bpec} - i_L + i_{o pv} + i_{o w}]^2
\end{align*}
\]

subject to

\[
\begin{align*}
0 \leq i_d &\leq i_{d_{\text{max}}} \\
0 \leq i_q &\leq i_{q_{\text{max}}} \\
0 \leq \omega_e &\leq \omega_{\epsilon_{\text{max}}} & \text{Upper bound} \\
0 \leq d_w &\leq 1 & \text{and}\ \\
0 \leq i_{pv} &\leq i_{pv_{\text{max}}} & \text{lower bound} \\
0 \leq v_{pv} &\leq v_{pv_{\text{max}}} & \text{inequality} \\
0 \leq d_{pv} &\leq 1 & \text{constraints} \\
0 \leq i_b &\leq i_{b_{\text{max}}} \\
0 \leq v_{DC} &\leq v_{gas}
\end{align*}
\]

\[
\begin{align*}
-\frac{P}{2} \omega_m i_d + \frac{P}{2} \omega_m i_q - \frac{\pi v_{DC}^2}{3\sqrt{3} \sqrt{i_d^2 + i_q^2}} \frac{1}{d_w} &= 0 \\
-\frac{3}{2} P \omega_m i_q + \frac{C_p\rho A V^3}{\omega_m} &= 0 \\
-C_p + \frac{1}{2} \left[ 116 \left( \frac{1}{\lambda} - 0.035 \right) - 5 \right] e^{-21 \left( \frac{1}{\lambda} - 0.035 \right)} &= 0 \\
-\lambda + \frac{r_{\text{p}}}{v_{pv}} &= 0 \\
-i_{o_{pv}} - \left[ n_p i_{ph} - n_p i_{rs} \left( e^{\frac{v_{pv}}{n_p K T}} - 1 \right) \right] &= 0 \\
-i_{ph} + \left[ S_{\text{scr}} + k_f (T - T_e) \right] \frac{S_{\text{scr}}}{100} &= 0 \\
-i_{rs} + i_{tr} \left[ \frac{T}{T_e} \right]^3 e^{\frac{v_{pv}}{n_p K T} \left( \frac{T}{T_e} - \frac{1}{2} \right)} &= 0 \\
v_{pv} - v_{DC} \frac{1}{d_{pv}} &= 0 \\
-i_{o_{pv}} + i_{pv} \frac{1}{d_{pv}} &= 0 \\
i_L + i_b - i_{o_{pv}} - i_{o_{w}} &= 0 \} \text{Power balance equation} \\
v_{DC} - e_{b} - i_b R_{i} &= 0 \} \text{Battery model equation}
\end{align*}
\]
The vector containing the problem’s design variables is

\[
x = \begin{bmatrix}
  i_d \\
  i_q \\
  \omega_m \\
  d_w \\
  i_{pe} \\
  v_{pe} \\
  d_{pv} \\
  i_b \\
  v_{DC}
\end{bmatrix}
\] (5.12)

where the actual decision variables are the control signals of the DC/DC converters \(d_w\) and \(d_{pv}\). However, these are also interrelated by the rest of the variables in vector \(x\).

Note that the objective function is non-linear, and that the model includes both linear and non-linear constraints. The upper and lower bound constraints of the form \(lb \leq x \leq ub\) are responsible for restricting the decision variables to practical and feasible values within the capabilities of the system. Similarly, the non-linear equality constraints are necessary for maintaining the power balance in the system.

The inputs to the problem are measurements of different parameters that dictate the operation mode of the system. The vector containing the problem’s inputs is denoted by

\[
u = \begin{bmatrix}
  V \\
  T \\
  S \\
  i_L \\
  e_b
\end{bmatrix}.
\] (5.13)
6. Problem Solution

The problem formulated in chapter 5 is a multi-objective optimization problem. These type of problems typically have conflicting objectives, such that a gain in one objective may negatively affect the other objective(s). Therefore the definition of optimality is not obvious. For this reason the decision maker must choose a solution based on experience or trade-off analysis.

The solution to these problems is approached using Pareto optimality techniques such as the weight method or the $\epsilon$-constraint method. Using these techniques, the problem may have an infinite number of efficient points that constitute a Pareto front curve (also called Pareto frontier or Pareto set) as illustrated in figure 6.1a.

The weight method is generally used when the objective functions form a convex Pareto frontier; the objective functions are not always required to be convex in order to create convex Pareto frontier. However, a multi-objective optimization problem is said to be convex if the feasible objective region is convex or if the feasible region is convex and the objective functions are quasiconvex with at least one quasiconvex function [23]. If the functions are non-convex then the $\epsilon$-constraint method is the preferred method between the two.

The two methods mentioned above are solved by scalarization. This means that the multi-objective problem is converted into a single objective function to find a solution that will minimize this single objective function while maintaining the physical constraints of the system [23].
Furthermore, the solutions obtained are normalized to be consistent with the weights assigned to the objective functions. The functions are normalized using the form

\[ 0 \leq \frac{f_i(x) - z_i^U}{z_i^N - z_i^U} \leq 1 \]  

where \( f_i(x) \) is the \( i^{th} \) objective function; \( z_i^U \) is the lower bound (Utopia point) of the Pareto set, normally infeasible because of the conflicting nature of the individual objectives; and \( z_i^N \) is the upper bound of the Pareto optimal set, obtained from the components of the Nadir point \( z^N \) [24].
6.1 Solution Approach

For this system’s study, both objective functions were evaluated using the
weight method, which resulted in a convex feasible objective region of the shape
shown in figure 6.1a. The weighted objective function is of the form

\[ f(x) = W_1 f_1(x) + W_2 f_2(x), \]  

(6.2)

where

\[ f_1(x) = THD_{i_w}(x)^2 = \sum_{n=1}^{\infty} \left( \frac{1}{\pi n} \sin(\pi n d_w) \right)^2 \]  

(6.3)

\[ f_2(x) = \Delta i_b = \left[ i_{b_{\text{spec}}} - i_L + \frac{i_{pv}}{d_{pv}} + \frac{\pi}{2\sqrt{3}} \sqrt{i_q^2 + i_d^2} \right] \frac{1}{d_w} \]  

(6.4)

and \( W_1 = 0, 0.01, 0.02, \ldots, 1 \) and \( W_2 = 1 - W_1 \).

The problem was solved using MATLAB’s \textit{fmincon} function. The code is
found in Appendix B and a description of the \textit{fmincon} function is given in
appendix C. The problem is initialized by means of a warm start iteration since
the problem solution does not change significantly from one iteration to another.
This process consists of using the solution of the previous iteration \( x_r^{-1} \) to set
the initial conditions for the new iteration \( r \).

As mentioned in chapter 5 the base for the formulation of the problem
focuses on mode 6 since it provides the most flexibility in terms of the power
distribution to the load and the batteries. Based on this presumption three
scenarios were analyzed:

- Scenario 1: \( \hat{i}_{o_w} + \hat{i}_{o_pv} \geq i_L + i_b \) where \( \hat{i}_{o_w} < i_L + i_b \) and \( \hat{i}_{o_pv} < i_L + i_b \);
- Scenario 2: $\hat{i}_{o,w} + \hat{i}_{o,pv} \geq i_L + i_b$ where $\hat{i}_{o,w} \geq i_L + i_b$ and $\hat{i}_{o,pv} < i_L + i_b$;

- Scenario 3: $\hat{i}_{o,w} + \hat{i}_{o,pv} \geq i_L + i_b$ where $\hat{i}_{o,w} < i_L + i_b$ and $\hat{i}_{o,pv} \geq i_L + i_b$;

where $\hat{i}_{o,w}$ and $\hat{i}_{o,pv}$ respectively are the wind and solar maximum available output currents. The voltage of the batteries determines the amount of charging current demand and therefore each scenario is analyzed with three different voltage levels:

- Completely discharged → 85% of nominal voltage;

- Partially charged → 92% of nominal voltage;

- Fully charged → 100% of nominal voltage.

### 6.1.1 Scenario 1

Table 6.1 includes the system’s inputs used to model Scenario 1. The Pareto sets for this scenario are shown in figure 7.5. Since the feasible region is convex the problem is suitable to be solved by the weighted method. The convex shape of this Pareto frontier illustrates the conflicting nature of the two objective functions for this scenario. If we choose to improve $\Delta i_b$, $THD_{i_w}$ would be worsened; similarly, if $THD_{i_w}$ is improved, $\Delta i_b$ is worsened.

<table>
<thead>
<tr>
<th>Scenario</th>
<th>$V$ [m/s]</th>
<th>$S$ [mW/cm²]</th>
<th>$T$ [K]</th>
<th>$\epsilon_b$ [Volts]</th>
<th>$i_L$ [Amperes]</th>
<th>$i_{b_{eq}}$ [Amperes]</th>
<th>$i_{a_w}$ [Amperes]</th>
<th>$\hat{i}_{o,w}$ [Amperes]</th>
<th>$\hat{i}_{o,pv}$ [Amperes]</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>7</td>
<td>60</td>
<td>298</td>
<td>40.8</td>
<td>30</td>
<td>12.0</td>
<td>22.13</td>
<td>20.94</td>
<td></td>
</tr>
<tr>
<td>1</td>
<td>7</td>
<td>60</td>
<td>298</td>
<td>44</td>
<td>30</td>
<td>6.66</td>
<td>20.55</td>
<td>19.44</td>
<td></td>
</tr>
<tr>
<td>1</td>
<td>7</td>
<td>60</td>
<td>298</td>
<td>48</td>
<td>30</td>
<td>4.0</td>
<td>18.84</td>
<td>17.81</td>
<td></td>
</tr>
</tbody>
</table>
\[ f_2(x) = \Delta i_b^2 \text{ [Amperes}^2] \]
\[ f_1(x) = \min_x \text{THD}_w(x)^2 \% \]

\( e_b = 40.8 \text{ Volts} \)

(a) \( f_1(x) \) vs \( f_2(x) \) for \( e_b = 40.8 \text{ Volts} \)

\( e_b = 44 \text{ Volts} \)

(b) \( f_1(x) \) vs \( f_2(x) \) for \( e_b = 44 \text{ Volts} \)

\( e_b = 48 \text{ Volts} \)

(c) \( f_1(x) \) vs \( f_2(x) \) for \( e_b = 48 \text{ Volts} \)

**Figure 6.2:** Scenario 1 Pareto set
Figure 6.3: $T_e$ and $i_{\omega w}$ as a function of $d_w$ for different wind velocities $V$
This conflict exists because $THD_{i_w}$ is minimized when $d_w = 100\%$ as shown in figure 6.4. Furthermore, figure 6.3 shows that the output current of the wind system $i_{o,w}$ and the electrical torque $T_e$ approach zero as $d_w$ approaches one. Since the solar system is not able to supply the load independent of the wind system, $i_{o,w}$ must be greater than zero in order to satisfy $\Delta i_b$, which creates a conflict between both objectives.

For this scenario minimizing $\Delta i_b$ is more critical than minimizing $THD_{i_w}$ because the power balance equality constraint $i_L + i_b - i_{o,w} - i_{o,pv} = 0$ must be satisfied. For this reason, more weight should be given to $\Delta i_b$ when choosing an optimal solution. Giving more weight to $THD_{i_w}$ would expose the batteries to undercharging. Considering the fact that the batteries could represent up to 40\% of the total cost of the system [11], the opportunity cost of giving more weight to $\Delta i_b$ is well justified.
6.1.2 Scenario 2

The inputs used to simulate scenario 2 are included in table 6.2. Figure 6.5 shows the Pareto set for scenario 2, which forms a convex Pareto frontier. Similar to scenario 1 the shape of this curve indicates a conflict between $THD_{i_w}$ and $\Delta i_b$.

<table>
<thead>
<tr>
<th>Scenario</th>
<th>$V$</th>
<th>$S$</th>
<th>$T$</th>
<th>$\epsilon_b$</th>
<th>$i_L$</th>
<th>$i_{b_{pv}}$</th>
<th>$i_{w_{pv}}$</th>
<th>$i_{w_{pv}}$</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>[m/s]</td>
<td>[mW/cm²]</td>
<td>[K]</td>
<td>[Volts]</td>
<td>[Amperes]</td>
<td>[Amperes]</td>
<td>[Amperes]</td>
<td>[Amperes]</td>
</tr>
<tr>
<td>2</td>
<td>9</td>
<td>60</td>
<td>298</td>
<td>40.8</td>
<td>20</td>
<td>12.0</td>
<td>47.36</td>
<td>20.94</td>
</tr>
<tr>
<td></td>
<td>9</td>
<td>60</td>
<td>298</td>
<td>44</td>
<td>20</td>
<td>6.66</td>
<td>43.42</td>
<td>19.44</td>
</tr>
<tr>
<td></td>
<td>9</td>
<td>60</td>
<td>298</td>
<td>48</td>
<td>20</td>
<td>4.0</td>
<td>39.86</td>
<td>17.81</td>
</tr>
</tbody>
</table>

In this case $THD_{i_w}$ is significantly less than the results obtained in scenario 1. This is because the amount of available wind power is large so the control signal $d_w$ does not need to be as small as in scenario 1. This situation allows the weight of $THD_{i_w}$ to be greater than the previous scenario, thus improving $THD_{i_w}$. Nevertheless, $\Delta i_b$ requires more weighing factor to ensure proper charging of the batteries.
(a) $f_1(x)$ vs $f_2(x)$ for $e_b = 40.8$ Volts

(b) $f_1(x)$ vs $f_2(x)$ for $e_b = 44$ Volts

(c) $f_1(x)$ vs $f_2(x)$ for $e_b = 48$ Volts

Figure 6.5: Scenario 2 Pareto set
6.1.3 Scenario 3

This scenario’s simulation inputs are depicted in table 6.3. In this case the objectives do not conflict with one another. This is because both objectives may be minimized at the same time since \( \hat{i}_{o, pv} > i_L + i_b \), allowing \( THD_{tw} \) and \( \Delta i_b \) to be minimized at all times.

**Table 6.3: Scenario 3 Inputs**

<table>
<thead>
<tr>
<th>Scenario</th>
<th>( V ) [m/s]</th>
<th>( S ) [mW/cm²]</th>
<th>( T ) [K]</th>
<th>( \epsilon_b )</th>
<th>( i_L ) [Amperes]</th>
<th>( i_{b,w} ) [Amperes]</th>
<th>( \dot{i}_{o, pv} ) [Amperes]</th>
<th>( \dot{i}_{o, pv} ) [Amperes]</th>
</tr>
</thead>
<tbody>
<tr>
<td>3</td>
<td>5</td>
<td>100</td>
<td>298</td>
<td>40.8</td>
<td>20</td>
<td>12.0</td>
<td>8.11</td>
<td>36.50</td>
</tr>
<tr>
<td>5</td>
<td>5</td>
<td>100</td>
<td>298</td>
<td>44</td>
<td>20</td>
<td>6.66</td>
<td>7.52</td>
<td>33.83</td>
</tr>
</tbody>
</table>

Since the objectives do not conflict with one another, for all cases when the solar system is able to fulfill the entire load independent of the wind system, the load will be supplied by the solar system only by setting \( d_w = 100\% \). This operation strategy ensures both functions to be minimized. Using the parameters established in table ?? we obtain the results from table 6.4. It is important to note that even though the current is very small, the rotor shaft speed is not zero. This is because there is available power from the wind turbine that is not being used. It is not desired to halt the machine because stress occurs at the surface of the shaft if the machine is started when the radial position is at rest as mentioned in chapter 3.
Table 6.4: Scenario 3 Formulation Results

<table>
<thead>
<tr>
<th>x</th>
<th>$(e_b = 40.8\text{ Volts})$</th>
<th>$(e_b = 44\text{ Volts})$</th>
<th>$(e_b = 48\text{ Volts})$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$i_d$ [Amperes]</td>
<td>0.00</td>
<td>0.00</td>
<td>0.00</td>
</tr>
<tr>
<td>$i_q$ [Amperes]</td>
<td>0.1</td>
<td>0.16</td>
<td>0.26</td>
</tr>
<tr>
<td>$\omega_m$ [rad/s]</td>
<td>6.32</td>
<td>6.73</td>
<td>7.31</td>
</tr>
<tr>
<td>$d_w$ [%]</td>
<td>100</td>
<td>100</td>
<td>100</td>
</tr>
<tr>
<td>$i_{pv}$ [Amperes]</td>
<td>16.14</td>
<td>16.27</td>
<td>16.28</td>
</tr>
<tr>
<td>$v_{pv}$ [Volts]</td>
<td>82.79</td>
<td>72.70</td>
<td>70.572</td>
</tr>
<tr>
<td>$d_{pv}$ [%]</td>
<td>50.6</td>
<td>61.3</td>
<td>68.5</td>
</tr>
<tr>
<td>$i_b$ [Amperes]</td>
<td>12.00</td>
<td>6.66</td>
<td>4.00</td>
</tr>
<tr>
<td>$v_{DC}$ [Volts]</td>
<td>41.90</td>
<td>44.61</td>
<td>48.36</td>
</tr>
</tbody>
</table>
7. Discussion

Different scenarios were analyzed to verify the performance of the optimization formulation. These scenarios are evaluated using high, average, and low values of various system’s inputs. The combinations are shown in table 7.1. The cell temperature $T$ is fixed to 298 K (25°C) for all scenarios and the objectives respective weights are $W_1 = 0.05$ and $W_2 = 0.95$, giving more weight to $\Delta i_b$ as explained in chapter 3. Since the number of evaluations resulting from these scenarios is quite large only a few scenarios were illustrated in this section. Nevertheless, the results of all evaluations correspondent to mode 6 of operation are shown in appendix D.

It is important to note that all cases from table 7.1 where $\hat{i}_{o,w} + \hat{i}_{o,v} < i_L + i_b$ do not correspond to mode 6 of operation and therefore are ignored. Table 7.2 shows all scenarios corresponding to mode 6 represented by “6” and all other modes of operation by “X”.

<table>
<thead>
<tr>
<th>Case</th>
<th>V [Volts]</th>
<th>S [mW/cm²]</th>
<th>$e_b$ [Volts]</th>
<th>$i_L$ [Amperes]</th>
</tr>
</thead>
<tbody>
<tr>
<td>Study LOW (˘)</td>
<td>5</td>
<td>20</td>
<td>40.8</td>
<td>10</td>
</tr>
<tr>
<td>AVERAGE ( = )</td>
<td>7</td>
<td>60</td>
<td>44</td>
<td>30</td>
</tr>
<tr>
<td>HIGH (ˆ)</td>
<td>10</td>
<td>100</td>
<td>48</td>
<td>60</td>
</tr>
</tbody>
</table>

Table 7.1: Evaluated Scenarios
Since the main objective of any electric power system is to serve the load, our system is mostly sensitive to changes in the load demand, more than any other system input. For this reason, it is important to illustrate the system response to changes in the load demand.

In figure 7.1 the objective functions show no conflict while $\hat{i}_{o,pv} \geq i_L + i_b$. However, when the load increases so that $\hat{i}_{o,w} < i_L + i_b$ and $\hat{i}_{o,pv} < i_L + i_b$ then $THD_{i_w}$ begins to increase and $\Delta i_b$ remains at zero. These characteristics are consistent with the control signals’ behavior against the load demand increments as shown in figure 7.2. Moreover, the wind turbine’s direct and quadrature axis currents approach zero as $d_w$ approaches one, and start increasing as $d_w$ decreases. The same behavior is expected from the rotor shaft speed.
\( \omega_e \), corroborating the results described in chapter 6 and table 7.2.

The solar system’s current \( i_{pv} \) changes abruptly with a small change on the control signal \( d_{pv} \). The voltage \( v_{pv} \) is high when the current demand is low and falls to the voltage which maximizes the power output of the solar system once the demand increases to that level. This is illustrated in figures 7.2b and 7.4.

![Graph](image-url)

- **(a) THD\(_{i_w} \) vs \( i_L \)**
- **(b) \( \Delta i_b \) vs \( i_L \)**

**Figure 7.1:** Objective functions variation with respect to \( i_L \)
Furthermore, the batteries current $i_b$ and voltage $v_{DC}$ remain constant since the batteries are assumed to be fully charged for this case study and $\Delta i_b = 0$ for all values of $i_L$.

![Graph](image)

(a) $d_w$ vs $i_L$

![Graph](image)

(b) $d_{pv}$ vs $i_L$

**Figure 7.2:** Control signals variation with respect to $i_L$
Figure 7.3: Wind system design variables variation with respect to $i_L$
Figure 7.4: Solar system design variables variation with respect to $i_L$
Figure 7.5: Battery system design variables variation with respect to $i_L$
8. Conclusion

This study has analyzed a stand-alone hybrid wind/solar system with energy storage. The subset of systems has been thoroughly researched and evaluated in order to model the system accurately. Special attention was given to the sensitivity of the equipment against potential operational disturbances in order to determine and protect against any life-shortening operational conditions. Eight modes of operation were defined and mode 6 of operation was analyzed in detail because this mode of operation is the most suitable for optimization.

The optimization was formulated as a multi-objective function with linear and non-linear constraints. Given that the objective space formed a convex set, the problem was solved using the weighted objective function method.

Three scenarios from operation mode 6 were analyzed. These scenarios result in conflicting objectives when the solar system may not supply the loads independent of the wind system. When the solar system may supply the loads without contribution from the wind, the load is supplied by the solar system only and no optimization is required.

This study’s research demonstrate that the lifespan of the equipment may be enhanced by reducing the operational disturbances in the system that shorten the equipment’s lifespan. The results presented in this study demonstrate that these operational disturbances may be reduced using the solution’s implementation and therefore the proposed formulation extends the lifespan of the equipment. This achievement is valuable because it provides an economical benefit.
to the system’s owner. This study proposes concepts that could be introduced in the economic dispatch optimization of wind and solar REEPS.

Moreover, wind and solar REEPS are typically part of the base load of the system in larger-scale power systems, which enables the flexibility to perform optimization techniques similar to those applied in mode 6 of this study. However, the application of these formulations into an actual system depends heavily on the proper operation of a custom-designed control system.

Finally, further improvements may be done by designing and implementing a robust controller that would merge the concepts introduced in this study along with the model referenced in [7]. Additionally, a real-life implementation of the system would provide more accurate results using real-time information and measurements.
APPENDIX A. Simulation Parameters

Battery system:
\[ C = 40 \text{ A-hr} \]
\[ e_b = 2.0 \text{ Volts} \]
\[ i_{b,max} = \frac{C}{4} \text{ Amps} \]
\[ m_c = 6 \]
\[ m_p = 10 \]
\[ m_s = 4 \]
\[ R_i = 0.022 \Omega \]
\[ v_{gas} = 2.4 \text{ Volts/cell} \]
\[ \xi = 85 \% \]

Wind system:
\[ A_w = 10.636 \text{ m}^2 \]
\[ C_p(\lambda_{opt}) = 0.382 \text{ at } \lambda_{opt} = 7.954 \]
\[ L_h = 3.55 \text{ mH} \]
\[ P = 28 \text{ Poles} \]
\[ r_s = 0.3676 \Omega \]
\[ R = 1.84 \text{ m} \]
\[ \vartheta = 0 \text{ degree} \]
\[ \lambda_{opt} = 7.954 \text{ at } \vartheta = 1 \]
\[ \rho = 1.225 \text{ kg/m}^3 \]
\[ \phi_m = 0.2867 \text{ Wb} \]
Solar system:

\[ A = 1.60 \]

\[ E_G = 1.10 \text{ Volts} \]

\[ i_{scr} = 3.27 \text{ Amps} \]

\[ i_{rr} = 2.0793 \times 10^{-6} \text{ A} \]

\[ k = 1.3805 \times 10^{-23} \text{ Nm/K} \]

\[ k_I = 0.0017 \text{ A/°C} \]

\[ n_p = 5 \]

\[ n_s = 200 \]

\[ q = 1.6 \times 10^{-19} \text{ C} \]

\[ T_r = 301.18 \text{ K} \]

System initial conditions:

\[ i_d = 1.61412 \]

\[ i_q = 11.395 \]

\[ \omega_m = 20 \]

\[ d_w = 0.3 \]

\[ i_{pv} = 15.436 \]

\[ v_{pv} = 96.00 \]

\[ d_{pv} = 0.3 \]

\[ i_b = 10 \]

\[ v_{DC} = 48 \]
APPENDIX B. Computational Code

B.1 Scenarios Generator

% Scenario parameters

v = [5 7 10];
v_text = strvcat('v_LOW', 'v_AVG', 'v_HIGH');
S_pv = [20 60 100];
S_pv_text = strvcat('S_pv_LOW', 'S_pv_AVG', 'S_pv_HIGH');
T = [273 273+25 273+50];
T_text = strvcat('T_LOW', 'T_AVG', 'T_HIGH');
eb_par = [0.85 0.9167 1];
eb_par_text = strvcat('e_b_LOW', 'e_b_AVG', 'e_b_HIGH');
i_L = [10 30 60];
i_L_text = strvcat('i_L_LOW', 'i_L_AVG', 'i_L_HIGH');

% Variables call function
S = Mejia_variables(S);

% Warm start initial conditions
S.Xin(1,:) = S.X_init;
S.Xin(2,:) = S.Xin(1,:);
S.Xin(3,:) = S.Xin(2,:);
S.Xin(4,:) = S.Xin(3,:);
S.X_init = S.Xin(1,:);

iter = 0;
count = 3;
for a = 1:1:3
    for b = 1:1:3
        for d = 1:1:3
            for e = 1:1:3
                S.Wind.v = v(a);
                S.Solar.S_pv = S_pv(b);
                S.Solar.T = 298;
                S.Batt.SOC = SOC(d);
                S.W1 = 0.05;
                S.W2 = 1-S.W1;
                S.i_L = i_L(e);
                S = Mejia_variables(S);
                S.X_init = S.Xin(count-2,:);
                S = Mejia_Solve_OPT(S);
                iter = iter+1;
                ss(iter,:) = [strvcat(v_text(a,:)) ' '...
                              strvcat(S_pv_text(b,:)) ' '...
                              strvcat(eb_par_text(d,:)) ' '...];
            end
        end
    end
end

56
strvcat(i_L_text(e,:)) ' '];
flag(iter,1) = S.exitflag;
xhist(iter,:) = S.X;
id = S.X(1);
ii = S.X(2);
we = S.X(3);
dw = S.X(4);
ipv = S.X(5);
vpv = S.X(6);
dpv = S.X(7);
ib = S.X(8);
Vb = S.X(9);
io_w = pi/(2*sqrt(3))*sqrt(ii^2+id^2)/dw;
io_pv = ipv/dpv;
x = [];
for ii = 1:1:50
  x(ii) = (2*iw_w*sin(pi*ii*dw)/(pi*ii))^2;
end
f1(iter,1) = (sum(x)/(2*dw*iw_w)^2);
f2(iter,1) = ((S.Batt.ib_spec-ib)^2);
S.Xin(count+1,:) = S.X;
count = count+1;
end end end

B.2 Optimization Solving Function
function S = Mejia_Solve_OPT(S)
%lower bound
LB = zeros(9,1);
%upper bound
UB = [Inf;Inf;Inf;1;Inf;Inf;1;S.Batt.ib_max;S.Batt.Vgas];
%Initial conditions
X_init = S.X_init;
%Linear inequality constraints
A = [];
b = [];
%Linear equality constraints
Aeq = [];
beq = [];
% Defines optimset
OptOpf = optimset('Display','iter','Diagnostics','on',...  
'DerivativeCheck','on','LargeScale','off','GradConstr',...
'off','GradObj','off','Hessian','off','MaxIter',1000,...'MaxFunEvals',1000);[X,fval,exitflag,output,lambda] = ...fmincon(@Mejia_OF,X_init,A,b,Aeq,beq,LB,UB,@Mejia_NLC,OptOpf,S);if exitflag<1 return endS.exitflag = exitflag;S.X = X;B.3 Variables Definition Functionfunction S = Mejia_variables(S)%System initial conditionsS.X_init_id = 1.6412;S.X_init_iq = 11.3951;S.X_init_we = 479.7681;S.X_init_dw = 0.22;S.X_init_ipv = 15.436;S.X_init_vpv = 96;S.X_init_dpv = 0.3;S.X_init_ib = 10;S.X_init_Vb = 48;S.X_init = [S.X_init_id;S.X_init_iq;S.X_init_we;S.X_init_dw;S.X_init_ipv;S.X_init_vpv;S.X_init_dpv;S.X_init_ib;S.X_init_Vb];%Battery system parameters
%Battery system constants
S.Batt.num_p = 10;
S.Batt.num_s = 4;
S.Batt.num_cell = 6;
S.Batt.eb = S.Batt.eb_per*(2*S.Batt.num_s*S.Batt.num_cell);
S.Batt.Vgas = 2.4*S.Batt.num_s*S.Batt.num_cell;
S.Batt.RI = 0.023*S.Batt.num_s;
S.Batt.C = 40;
S.Batt.ib_trickle = S.Batt.C*S.Batt.num_p/100;
if S.Batt.C*S.Batt.num_p*0.2*(1-S.Batt.eb_per)>S.Batt.ib_trickle
    S.Batt.ib_max = S.Batt.C*S.Batt.num_p*0.2*(1-S.Batt.eb_per);
else
    S.Batt.ib_max = S.Batt.ib_trickle;
end

if 0<= S.Batt.eb && (48-S.Batt.ib_max*S.Batt.RI)>S.Batt.eb
    S.Batt.ib_spec = S.Batt.ib_max;
elseif (48-S.Batt.ib_max*S.Batt.RI)< = S.Batt.eb && 48*0.95>S.Batt.eb
    S.Batt.ib_spec = (48-S.Batt.eb)*S.Batt.num_p/S.Batt.RI;
elseif 48*0.95< = S.Batt.eb && S.Batt.Vgas>S.Batt.eb
    S.Batt.ib_spec = S.Batt.ib_trickle;
else
    S.Batt.eb> = S.Batt.Vgas
    S.Batt.ib_spec = 0;
end

%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
%%%%%%%%%%%%%%%% Solar system parameters %%%%%%%%%%%%%%%%%%%%%%
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%

% Solar system variables
%   S.Solar.T = 25+273;
%   S.Solar.S_pv = 100;

% Solar system constants
S.Solar.n_p = 5;
S.Solar.n_s = 200;
S.Solar.q = 1.6*10^-19;
S.Solar.A = 1.6;
S.Solar.k = 1.3805*10^-23;
S.Solar.I_scr = 3.27;
S.Solar.k_t = 0.0017;
S.Solar.I_rr = 2.0793*10^-6;
S.Solar.T_r = 301.18;
S.Solar.E_G = 1.10;
% Wind system parameters

% Wind system variables
S.Wind.v = 14;
S.Wind.we = 479.768;

% Wind system constants
S.Wind.rs = 0.3676;
S.Wind.L = 0.00355;
S.Wind.phi = 0.2867;
S.Wind.rho = 1.225;
S.Wind.BPA = 1;
S.Wind.P = 28;
S.Wind.C1 = 0.5;
S.Wind.C2 = 116;
S.Wind.C3 = 0.4;
S.Wind.C4 = 0;
S.Wind.C5 = 5;
S.Wind.C6 = 21;
S.Wind.y = 2;
S.Wind.r = 1.84;
S.Wind.A = pi*S.Wind.r^2;
S.Wind.v_cutin = 3;
S.Wind.v_cutout = 20;

B.4 Objective Function
function f = Mejia_OF(X,S)

% Decision variables
S.Wind.id = X(1);
S.Wind.iq = X(2);
S.Wind.we = X(3);
S.Wind.dw = X(4);
S.Solar.ipv = X(5);
S.Solar.vpv = X(6);
S.Solar.dpv = X(7);
S.Batt.ib = X(8);
S.Batt.Vb = X(9);

% Wind system output current
io_w = pi/(2*sqrt(3))*sqrt(S.Wind.iq^2+S.Wind.id^2)/S.Wind.dw;

% Solar system output current
io_pv = S.Solar.ipv/S.Solar.dpv;

% Sum of the vectorized wind system current
x = [];
for ii = 1:50
    x(ii) = \((2*io_w*\sin(\pi*ii*S.Wind.dw)/(\pi*ii))^2;\)
end

%Objective functions for weighted objectives
f1 = \((\text{sum}(x)/(2*S.Wind.dw*io_w)^2);\)

f2 = \((S.Batt.ib\_spec-S.Batt.ib)^2;\)

f = S.W1*f1 + S.W2*f2;

B.5 Non-linear Constraints Function
function [C, Ceq] = Mejia_NLC(X,S)

%Decision variables
S.Wind.id = X(1);
S.Wind.iq = X(2);
S.Wind.wm = X(3);
S.Wind.dw = X(4);
S.Solar.ipv = X(5);
S.Solar.vpv = X(6);
S.Solar.dpv = X(7);
S.Batt.ib = X(8);
S.Batt.Vb = X(9);

%Wind system parameters
S.Wind.lambda = S.Wind.r*S.Wind.wm/S.Wind.v;
S.Wind.lambda\_ref = 1/(S.Wind.lambda+0.08*S.Wind.BPA)-... 0.035/(S.Wind.BPA-3+1);

io_w = \(\pi/(2*sqrt(3))*sqrt(S.Wind.iq^2+S.Wind.id^2)/S.Wind.dw;\)

%Solar system parameters


io_pv = S.Solar.ipv/S.Solar.dpv;

%Inequality constraint
C = [];

%Equality constraints
Ceq = [-S.Wind.rs*S.Wind.iq/S.Wind.L-...
clear all
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clc

%S System Inputs
S.Batt.eb_per = 0.85;
S.Wind.v' = 14;
S.Solar.S_pv = 100;
S.Solar.T = 298;
S.i_L = 10;
S = Mejia_variables(S);
S = Mejia_Solve_OPT(S);
S.Xin(1,:,:) = S.X_init;
S.Xin(2,:) = S.Xin(1,:);  
S.Xin(3,:) = S.Xin(2,:);  
S.Xin(4,:) = S.Xin(3,:);  
S.X_init = S.Xin(1,:);
jter = 0;
iter1 = 0;
for ii = 0:1:60
  iter1 = iter1+1;
  iter2 = 0;
  count = 2;
  for y = 0:0.01:1
    S.W1 = y;
    S.W2 = 1-S.W1;
S.X_init = S.Xin(count-1,:);
S = Mejia_Solve_OPT(S);
iter = iter+1;
iter2 = iter2+1;
xhist(iter,:) = S.X;
id = S.X(1);
iq = S.X(2);
wm = S.X(3);
dw = S.X(4);
ipv = S.X(5);
vpv = S.X(6);
dpv = S.X(7);
ib = S.X(8);
Vb = S.X(9);
io_w = pi/(2*sqrt(3))*sqrt(iq^2+id^2)/dw;
io_pv = ipv/dpv;
x = [];
for ii = 1:1:50
    x(ii) = (2*iow*sin(pi*ii*dw)/(pi*ii))^2;
end
f1(iter2,iter1) = (sum(x)/(2*dw*iow)^2);
f2(iter2,iter1) = ((S.Batt.ib_spec-ib)^2);
flag(iter,1) = S.exitflag;
S.Xin(count+1,:) = S.X;
count = count+1;
end
f1min = min(f1);
f1max = max(f1);
f2min = min(f2);
f2max = max(f2);
f1norm = (f1-f1min)/(f1max-f1min);
f2norm = (f2-f2min)/(f2max-f2min);
APPENDIX C. MATLAB’s \textit{fmincon} Function

MATLAB’ \textit{fmincon} function is designed to find the minimum of a constrained non-linear multi-variable function. MATLAB solves and optimization problem of the form

$$\min_x f(x)$$

subject to

$$c(x) \leq 0$$

$$ceq(x) = 0$$

$$A \cdot x = beq$$

$$lb \leq x \leq ub$$

where \(x, b, beq, lb,\) and \(ub\) are vectors, \(A\) and \(Aeq\) are matrices, \(c(x)\) and \(ceq(x)\) are functions that return vectors (@nonlincon), and \(f(x)\) is a function that returns a scalar (@fun).

The function is defined in the MATLAB environment as

$$[X,\text{fval,exitflag,output,lambda}] = \text{fmincon}(@\text{fun},x0,A,b,Aeq,beq,lb,ub,@\text{nonlincon},\text{options})$$

where \(x0\) sets the initial conditions; \(X\) is the optimized value obtained from the function @\text{fun} subject to linear inequalities \(A x = beq\), non-linear inequalities \(c(x) \leq 0\), or equalities \(ceq(x) = 0\), in the range \(lb \leq x \leq ub\). Further, “\text{lambda}” is a structure containing the Lagrangean multipliers at the solution \(x\); “\text{exitflag}” is an integer identifying the reason the algorithm terminated; “\text{output}” is a structure containing information about the optimization, and \(\text{fval}\) contains the value of the objective function @\text{fun} at the solution \(X\).
APPENDIX D. System Case Studies

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Notes: 
- ΔiL: Δi refers to the change in load current due to varying wind speeds. 
- Flag: Indicates the type of flag or condition that governs the operation of the system. 
- uP, p: Represents the peak voltage level at which the system operates.

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BIBLIOGRAPHY


