Nicole King
History of Math
Question #22

**How were the Operation Signs Developed (+, -, X, /)?**

**Why are X, Y, Z Used For Unknown Variables?**

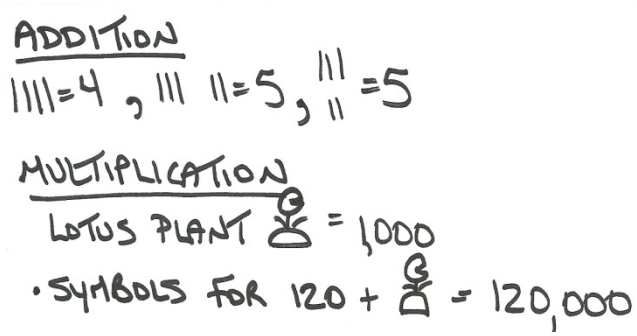
## What's Up With Notation?
## A Brief History of Mathematical Operations

"Every meaningful mathematical statement can also be expressed in plain language. Many plain-language statements of mathematical expressions would fill several pages, while to express them in mathematical notation might take as little as one line. One of the ways to achieve this remarkable compression is to use symbols to stand for statements, instructions and so on."
Lancelot Hogben

### Egyptian Notation 3300 B.C.

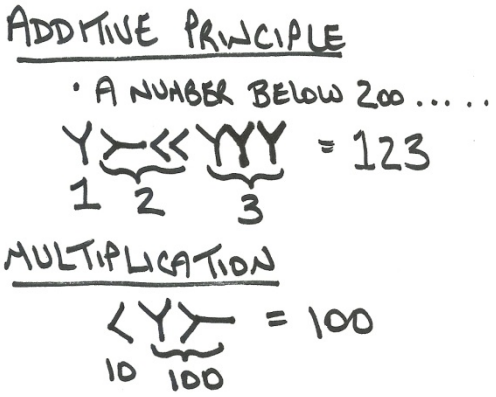
There is some symbolism in Egyptian algebra. In the Rhind papyrus we find symbols for plus and minus. The first of these symbols represents a pair of legs walking from right to left, the normal direction for Egyptian writing, and the other a pair of legs walking from left to right, opposite to the direction for Egyptian writing.

- In writing numbers, the Egyptians used the principles of addition and multiplication.
- In applying the additive principle, not more than 4 symbols of the same kind were placed in any one group.



## Babylonian Notation 2400 B.C.

- Originally Babylonia used placement of #'s to indicate addition and multiplication (There was a limited use of subtraction)
- Number's below 200 were expressed ordinarily by symbols whose respective values were to be added.
- The principle of multiplication reveals itself where the smaller symbol 10, placed before the 100, is to be multiplied by 100 so that this symbolism designates 1000.

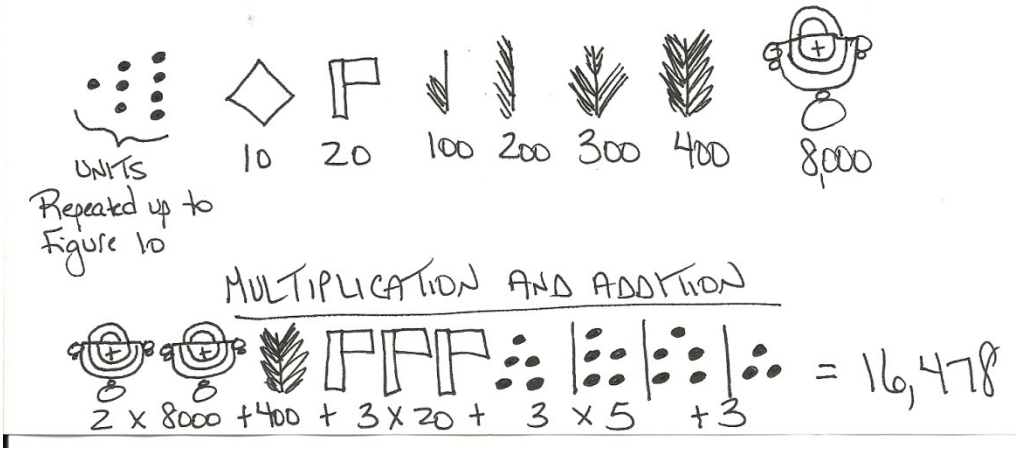



## Roman Notation- 500 B.C.

- The dominating feature is the principle of addition
- Addition = III (1 + 1 + 1 = 3) , XII (5 + 1 + 1 = 7), CC (100 + 100 = 200)
- There was frequent use of the principle of subtraction. If a letter is placed before another of greater value, its value is to be subtracted from that of the greater.
- Subtraction = IV (5-1 = 4) IX = (10 -1 = 9)

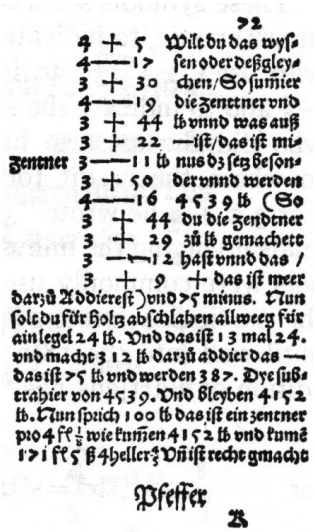
## Aztec Notation – 250 A.D.

- In the juxtaposition of symbols the additive principle is employed.
- For figures, one of the numerical signs was the dot (●)which marked the units and which was repeated either up to 20 or up to the figure 10, represented by a lozenge. The number 20 was represented by a flag, which, repeated 5 times gave the number 100.



## How were the (operation) signs developed? (+, -, X, /)?

### *Addition and Subtraction*




- The expression "plus or minus" is very old, having been in common use by the Romans to indicate simply "more or less". It is often found on Roman tombstones, where the age of the deceased is given as illustrated to indicate "94 years, more or less".

AN · $\overline{\text{LXXXX IIII}}$ · P · M

- In Europe in the early 15th century the letters "P" and "M" were generally used. The symbols (P with stroke for piu, i.e. plus, and M with stroke for meno, i.e. minus) appeared for the first time in Luca Pacioli's mathematics compendium, *Summa de Arithmetica, Geometria, Proportioni et Proportionalita*, first printed and published in Venice in 1494.

- They appeared in Johann Widman's (c1460-?) arithmetic published in Leipzig in 1489, the author saying: "Was - ist / das ist minus...vnd das + das ist mer." He then speaks of "4 centner + 5 pfund," and also of "4 centner - 17 pfund," thus showing the excess or deficiency in the weight of boxes or bales.

- The first one to make use of these signs in writing an algebraic expression was the Dutch mathematician Vander Hoecke, who in 1514 gave this illustration (on the left) for radical three quarters minus radical three fifths, and for radical 3 add 5 he gave the sign as shown on the right.

$\mathrm{R}\overline{\mathrm{x}}\,3 + 5$

- Robert Recorde, introduced plus and minus to Britain in 1557 in *The Whetstone of Witte*. In it he introduced these newfangled signs: "There be other 2 signes in often use of which the first is made thus + and betokeneth more: the other is thus made - and betokeneth lesse".

$\mathrm{R}\overline{\mathrm{x}}\,\frac{3}{4} - \mathrm{R}\overline{\mathrm{x}}\,\frac{3}{5}$

### *The Symbol for Addition*

+ —

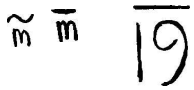
- Nicole d' Oresme (1323-1382) may have used a figure which looks like a plus symbol as an abbreviation for the Latin *et* (meaning "and") in *Algorismus proportionum,* believed to have been written between 1356 and 1361.
- In a manuscript of 1456, written in Germany, the word "et" is used for addition and is generally written so that it closely resembles the modern symbol for addition. There seems little doubt that the sign is merely a ligature for "et", much in the same way that we have the ligature "&" for the word "and."

- In the 16th century the Latin races generally followed the Italian school, using the letters p and m, each with the bar above it, or their equivalents, for plus and minus.

## The Symbol for Subtraction

- The use of the word minus as indicating an operation occurred in the works of Fibonacci (c1175-1250). In 1202. The bar above the letter simply indicated an omission. In the 15th century, this third symbol was also often used for minus, but most writers preferred the other variations.

- The origin of the minus sign has been more of a subject of dispute. Some have thought that it is a survival of the bar above the three symbols for minus as listed above. It is more probably that it comes from the habit of early scribes of using it as a shorthand equivalent of "m."



## ***The Symbol for Multiplication***

### The cross (X)

- William Oughtred (1574-1660) contributed vastly to the propagation of mathematical knowledge in English.

- Oughtred laid extraordinary emphasis upon the use of mathematical symbols : altogether he used over 150 of them.

- Only three have come down to modern times, namely the cross symbol for multiplication.

### The dot (•)

- Leibniz (1646-1715) objected to the use of Oughtred's cross symbol because of possible confusion with the letter X. On 29 July 1698 he wrote in a letter to John Bernoulli : "I do not like (the cross) as a symbol for multiplication, as it is easily confounded with x; .... often I simply relate two quantities by an interposed dot and indicate multiplication by ZC.LM."

- Through the aid of Christian Wolf (1679-1754) the dot was generally adopted in the 18th century as a symbol for multiplication.

### The asterisk (*)

- Used by Johann Rahn (1622-1676) in 1659 in *Teutsche Algebra*.

**By juxtaposition.**

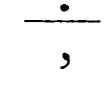
- In a manuscript found buried in the earth near the village of Bakhshali, India, and dating to the eighth, ninth, or tenth century, multiplication is normally indicated by placing numbers side-by-side.

- Multiplication by juxtaposition is also indicated in "some fifteenth-century manuscripts." Juxtaposition was used by al-Qalasadi in the fifteenth century

- The common symbol as illustrated was developed in England about 1600. It was not a new sign, having long been used in cross multiplication.

- The symbol was not readily adopted by arithmeticians, being of no practical value to them. In the 18th century some use was made of it in numerical work, but it was not until the second half of the 19th century that it became popular in elementary arithmetic. On account of its resemblance to x it was not well adapted to use in algebra, and so the dot came to be employed.

## *The Symbol for Division*

- Originally this sign (or a plain line) was used in ancient manuscripts to mark passages that were suspected of being corrupted or spurious.
- It was used as early as the 10th century for the word est. When written after the letter "i", it symbolized "id est." When written after the word "it", it symbolized "interest." If written after the word "divisa", for "divisa est", this might possibly have suggested its use as a symbol of division. 
- Around the year 1200, both the Arabic writer al-Hassar, and Fibonacci (Leonardo of Pisa), symbolised division in fraction form with the use of a horizontal bar, but it is thought likely that Fibonacci adopted al-Hassar's introduction of this symbolisation.
- David E. Smith writes, "It is impossible to fix an exact date for the origin of our present arrangement of figures in long division, partly because it developed gradually"

**Close parenthesis )**

- The arrangement 8)24 was used by Michael Stifel (1487-1567 or 1486-1567) in *Arithmetica integra,* which was completed in 1540 and published in 1544 in Nuernberg (Cajori vol. 1, page 269; DSB).

**The obelus (÷)**

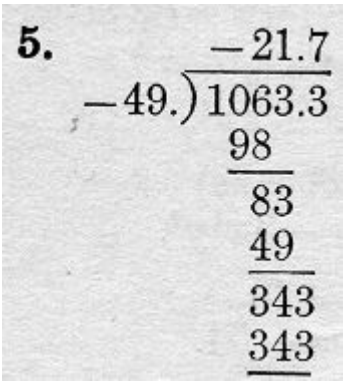
- First used as a division symbol by Johann Rahn (or Rhonius) (1622-1676) in 1659 in *Teutsche Algebra*.

**The colon (:)**

- Used in 1633 in a text entitled *Johnson Arithmetik; In two Bookes.* However Johnson only used the symbol to indicate fractions (for example three-fourths was written 3:4); he did not use the symbol for division "dissociated from the idea of a fraction" (Cajori vol. 1, page 276).

- Gottfried Wilhelm Leibniz (1646-1716) used : for both ratio and division in 1684 in the *Acta eruditorum* .

- In 1888 in the teacher's edition of *The Elements of Algebra* by G. A. Wentworth the vinculum is almost attached to the top of the close parenthesis and the quotient is written above the vinculum.




## Why are x,y,z used for unknown variables?

- Arabic algebra texts such as the *Al-Jabr* still described mathematics in full text rather than symbolic formulas, but denoted the variable quantity in Arabic word *šay'* شَيْء = "thing";. This was taken into Old Spanish with the pronunciation "šei", which was written *xei*, and was soon habitually abbreviated to ***X***; and this is still the customary variable name in many fields today. It started the habit of using letters to represent variables in algebra. Beyond mathematics, "X" has come to represent a generic placeholder variable whose value is unknown or secret.

**Greek letters.**

- The use of letters to represent general numbers goes back to Greek antiquity. Aristotle frequently used single capital letters or two letters for the designation of magnitude or number

- Diophantus (fl. about 250-275) used a Greek letter with an accent to represent an unknown. Nesselmann takes this symbol to be the final sigma and remarks that probably its selection was prompted by the fact that it was the only letter in the Greek alphabet which was not used in writing numbers. However, differing opinions exist
- Jordanus Nemorarius (1225-1260) used letters to replace numbers.
- In 1591 Francois Vieta (1540-1603) was the first person to use letters for unknowns and constants in algebraic equations. He used vowels for unknowns and consonants for given numbers (all capital letters) in *In artem analyticem isogoge.* Vieta wrote:

**Descartes' use of *z, y, x.***

- Used by early German writers, ȝe looked so much like an *x* that it could easily have been taken as such, and that Descartes actually did interpret and use it as an *x.*

- The use of *z, y, x* . . . to represent unknowns is due to René Descartes, in his *La géometrie* (1637). Without comment, he introduces the use of the first letters of the alphabet to signify *known* quantities and the use of the last letters to signify *unknown* quantities.

- The predominant use of the letter *x* to represent an unknown value came about in an interesting way. During the printing of *La Geometrie* and its appendix, *Discours de La Methode,* which introduced coordinate geometry, the printer reached a dilemma. While the text was being typeset, the printer began to run short of the last letters of the alphabet. He asked Descartes if it mattered whether *x, y,* or *z* was used in each of the book's many equations. Descartes replied that it made no difference which of the three letters was used to designate an unknown quantity. The printer selected *x* for most of the unknowns, since the letters *y* and *z* are used in the French language more frequently than is *x.*

### *Conclusion*

As an aside, when people began to write computer languages from the 1950s on, they were hampered by the absence of mathematical symbols in the character sets of the time: they had the plus, minus and equals signs, but not those for multiplication or division. So computer scientists had to improvise by borrowing the asterisk for multiplication and the forward slash for division

Like the words in our language, the signs for arithmetic have been invented and have become standard through historical accident, influenced by a very few pioneers.

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http://jeff560.tripod.com/operation.html

**Why I picked this question and it's relation to  middle school math curriculum:**

I believe these questions relate to middle school (and high school) math curriculum because these are questions students often have for me.  Now I will have answers.  What I found most interesting about the questions are that the symbols we use today are not only still evolving, but they seem to have come about by accident.  I believe that the answers to these questions will demonstrate for students that we are not done creating the language of math.