Date due: February 11, 2011

Note: If there is no designation, both grads and undergrads are to complete the problem. Each problem is of equal weight and the points you score are taken out of the total possible and allocated to 1/4 of the total points associated with assignment problems (since there are four assignments).

1. Capital Investment (Harvey Greenberg) - For the general model, prove that an optimal investment policy is to let \( x_{j^*} = b \) and \( x_j = 0 \) for \( j \neq j^* \), where \( j^* \) satisfies

\[
c_{j^*} = \max_j \{c_j\}, j = 1, \ldots, N
\]

2. Production (Harvey Greenberg)

(a) Solve the production problem using MATLAB’s Optimization Toolbox or LINGO/LINDO.

(b) How does the solution change if the unit cost to increase production is increased to 2.5 or decreased to 1.5? Argue empirically (undergrads) and analytically (grads).

3. Maximum Flow (Harvey Greenberg)

(a) (undergrads) Prove that an optimal solution exists for the example.

(b) (grads) Prove that an optimal solution exists for the general problem.

(c) Solve the example problem using MATLAB or LINGO/LINDO.

4. Transportation (Harvey Greenberg)

(a) (undergrads) Prove there is an optimal solution for the example.

(b) (grads) Prove that there is an optimal (hence feasible) solution to the general problem if and only if

\[
\sum_{i=1}^{m} a_i \geq \sum_{j=1}^{n} b_j
\]

(c) (grads) Prove that if

\[
\sum_{i=1}^{m} a_i = \sum_{j=1}^{n} b_j
\]
then every feasible policy must satisfy

$$
\sum_{j=1}^{n} x_{ij} = a_i, \quad i = 1, \ldots, m, \\
\sum_{i=1}^{m} x_{ij} = b_j, \quad j = 1, \ldots, n.
$$

5. Diet Problem (Harvey Greenberg)

(a) (grads) Prove that a solution exists for the example problem (under-grads), for the general problem.

(b) Use MATLAB or LINGO/LINDO to solve the example problem.

(c) For the example problem, look at the solution. Which nutrient requirements are not binding?

(d) (grads) How can we place a dollar value on each nutrient?

(e) (grads) Suppose food $j$ is used in an optimal solution. Prove that we can replace each dollar expenditure of food $j$ by $\lambda_1$ dollars of food 1, $\lambda_2$ dollars of food 2, $\lambda_r$ dollars of food $r$, provided

$$
\lambda_k \geq 0, \quad \sum_{k=1}^{r} \lambda_k = 1
$$

$$
\sum_{k=1}^{r} \lambda_k a_{ik} \geq a_{ij}, \quad \text{for all } i = 1, \ldots, m
$$

6. (grads) Prove 1-5 of "Some Facts" (section 1.2.3) in the section 1.2 of "Optimization Models" of the classnotes.

7. Solve, in MATLAB or LINGO/LINDO problems 3, 7, and 8 of Chapter 1 of our text.