Instructions: Please solve the following five problems. You are allowed to use any handwritten solutions to your other assignments but you must show all relevant work for full credit. Each problem is worth 2 points for a total of 10 points on this quiz. Good Luck!

1. A cyclist rides down a long straight road at a velocity (in m/min) given by \( v(t) = 400 - 20t \), for \( 0 \leq t \leq 10 \) min. How far has the cyclist traveled when her velocity is 200 m/min?

   - First solve for \( t \): \( v(t) = 400 - 20t = 200 \Leftrightarrow t = 10 \) min
   - Then find displacement: \( s(10) = \int_{0}^{10} (400 - 20t) \, dt = [400t - 10t^2]_{0}^{10} = 4000 - 1000 = 3000 \text{ m} \)

2. Review Questions

(a) Sketch a case in which the area bounded by two curves can only be found by integrating with respect to \( y \)? See book.

(b) What is the result of integrating a population growth rate between two times \( t = a \) and \( t = b \)? If \( Q'(t) \) is the growth rate of a population \( Q \) at time \( t \), then \( \int_{a}^{b} Q'(t) \, dt = Q(b) - Q(a) \) is the net change of the population over the period \([a, b]\\)

3. Explain why or why not.

(a) A particular marginal cost function has the property that it is positive and decreasing. The cost of increasing production from \( A \) units to \( 2A \) units is greater than the cost of increasing production from \( 2A \) units to \( 3A \) units. True. The cost of increasing production from \( A \) to \( B \) is given by \( \int_{A}^{B} C'(t) \, dt \), which is geometrically the area under the curve \( y = C'(x) \) from \( A \) to \( B \). If \( C' \) is positive and decreasing, then there is more area under the curve from \( A \) to \( 2A \) than from \( 2A \) to \( 3A \).

(b) Without evaluating integrals, \( \int_{0}^{1} (x - x^2) \, dx = \int_{0}^{1} (\sqrt{y} - y) \, dy \). True. They both represent the area in the first quadrant bounded by \( y = x \) and \( y = x^2 \) (on left side with respect to \( x \), on right side with respect to the inverses and \( y \)).

4. Find the area of the region bounded by \( y = \ln(x) \), \( y = 2 \), \( y = 0 \), and \( x = 0 \) (see Figure 1.1).

\[
A = \int_{0}^{e^{2}} dy = [e^{y}]_{0}^{2} = e^{2} - 1
\]

5. Suppose the power function of a large city over a 24-hr period is given by \( P(t) = E'(t) = 300 - 200 \sin(\frac{\pi t}{12}) \) where \( P \) is measured in MW and \( t = 0 \) corresponds to 6:00 p.m. (see Figure 1.2).

(a) How much energy is consumed by this city in the typical 24-hr period?

\[
E = \int_{0}^{24} 300 - 200 \sin \left( \frac{\pi t}{12} \right) \, dt = \int_{-12}^{12} 300 - 200 \sin \left( \frac{\pi t}{12} \right) \, dt \quad \text{using periodicity of given power function}
\]

\[
= 300 \int_{-12}^{12} 1 \, dt - 200 \int_{-12}^{12} \sin \left( \frac{\pi t}{12} \right) \, dt = 300 \cdot 12 - 200 \int_{-12}^{12} \sin \left( \frac{\pi t}{12} \right) \, dt \quad \text{using symmetry of (even) sine function}
\]

\[
= 300[1]_{-12}^{12} = 300 \cdot (12 - (-12)) = 300 \cdot 24 = 7200 \text{ MWh}
\]

(b) A typical wind turbine can generate electricity at a rate of about 200 kW. Approximately how many wind turbines are needed to meet the average energy needs of the city?

\[
\frac{7.2 \times 10^6 \text{ kWh/day}}{(200 \text{ kW/turbine})(24 \text{ hours/day})} = 1500 \text{ turbines}
\]