• Section 12.1: #24, #32, #40, #60

12.1.24: For the following sets of planes, determine which pairs of planes in the set are parallel, orthogonal, or identical.

\[ Q : x + y - z = 0 \quad R : y + z = 0 \quad S : x - y = 0 \quad T : x + y + z = 0 \]

The normal vectors for each of these planes are:

\[ \mathbf{n}_Q = \langle 1, 1, -1 \rangle, \quad \mathbf{n}_R = \langle 0, 1, 1 \rangle, \quad \mathbf{n}_S = \langle 1, -1, 0 \rangle, \quad \mathbf{n}_T = \langle 1, 1, 1 \rangle \]

It is clear that none of the normals are parallel to each other since none of them are scalar multiple of the others.

Taking pairwise dot products we see that \( \mathbf{n}_Q \perp \mathbf{n}_R, \mathbf{n}_Q \perp \mathbf{n}_S, \) and \( \mathbf{n}_S \perp \mathbf{n}_R. \) This means that \( Q \) is orthogonal to \( R \) and \( S \) and also \( S \) is orthogonal to \( R. \) None of the planes are identical.

12.1.32: Find the equation of the line where the planes \( Q \) and \( R \) intersect.

\[ Q : x - y - 2z = 1 \quad R : x + y + z = -1 \]

**Soln:** To find the line where the two planes intersect we first need to find the cross product of the normals. This will point in the direction of the line. Then we need a single point on the line. This allows us to parameterize the line of intersection.

\[ \mathbf{n}_Q = \langle 1, -1, -2 \rangle, \quad \text{and} \quad \mathbf{n}_R = \langle 1, 1, 1 \rangle \]

Therefore,

\[ \mathbf{n}_Q \times \mathbf{n}_R = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 1 & -1 & -2 \\ 1 & 1 & 1 \end{vmatrix} = \langle -1 + 2, -1 + 2, 1 + 1 \rangle = \langle 1, -3, 2 \rangle \]

To find a point on the line we’ll consider the point where the line crosses the \( xy \)-plane \((z = 0)\). This gives the system of equations

\[ \begin{cases} x - y = 1 \\ x + y = -1 \end{cases} \implies x = 0 \text{ and } y = -1 \]

Therefore, the point \((0, -1, 0)\) is on both planes and therefore on the line of intersection. Hence, the line can be written as

\[ \mathbf{r}(t) = \langle 1, -3, 2 \rangle t + \langle 0, -1, 0 \rangle = \langle t, -3t - 1, 2t \rangle \]

12.1.40: (a) Find the intercepts with the three coordinate axes, (b) find the equations of the \( xy \), \( xz \), and \( yz \) traces, (c) sketch a graph of the surface

\[ \frac{x^2}{6} + 24y^2 + \frac{z^2}{24} - 6 = 0 \]
Soln: First observe that we can rewrite the equation as

\[
\frac{x^2}{36} + 4y^2 + \frac{z^2}{144} = 1 \iff \frac{x^2}{36} + \frac{y^2}{(1/4)} + \frac{z^2}{144} = 1
\]

You should recognize this as an ellipsoid.

(a) \( x \)-intercept: \( (y = z = 0) \) \( x = \pm 6 \)
    \( y \)-intercept: \( (x = z = 0) \) \( y = \pm (1/2) \)
    \( z \)-intercept: \( (x = y = 0) \) \( z = \pm 12 \)

(b) The equations for the traces can be found by setting one variable at a time to zero.
    \( xy \)-trace: \( (z = 0) \) \( \frac{x^2}{36} + \frac{y^2}{(1/4)} = 1 \)
    \( xz \)-trace: \( (y = 0) \) \( \frac{x^2}{36} + \frac{z^2}{144} = 1 \)
    \( yz \)-trace: \( (x = 0) \) \( \frac{y^2}{(1/4)} + \frac{z^2}{144} = 1 \)

(c) See image at right.

12.1.60: (a) Find the intercepts with the three coordinate axes, (b) find the equations of the \( xy \), \( xz \), and \( yz \) traces, (c) sketch a graph of the surface

\[-\frac{x^2}{6} - 24y^2 + \frac{z^2}{24} - 6 = 0\]

Soln: Again observe that the function can be rewritten as

\[-\frac{x^2}{36} - \frac{y^2}{(1/4)} + \frac{z^2}{144} = 1\]

Recognize this as a hyperboloid with two sheets (two minus signs)

(a) \( x \)-intercept: \( (y = z = 0) \) none
    \( y \)-intercept: \( (x = z = 0) \) none
    \( z \)-intercept: \( (x = y = 0) \) \( z = \pm 12 \)

(b) The equations for the traces can be found by setting one variable at a time to zero.
    \( xy \)-trace: \( (z = 0) \) \[-\frac{x^2}{36} - \frac{y^2}{(1/4)} = 1 \]
    THIS IS IMPOSSIBLE \( \iff \) no \( xy \)-trace
    \( xz \)-trace: \( (y = 0) \) \[-\frac{x^2}{36} + \frac{z^2}{144} = 1 \]
    \( yz \)-trace: \( (x = 0) \) \[-\frac{y^2}{(1/4)} + \frac{z^2}{144} = 1 \]

(c) Hyperboloid with two sheets (see image at right)
Section 12.2: #18, #28, #32, #34, #59 (Use wolfram alpha or some other grapher to approximate the local minimum and maximum in problem 59. Provide a printout of the plot.)

12.2.18: Find the domain of
\[ h(x, y) = \sqrt{x - 2y + 4} \]

**Soln:** The argument of the square root must be non-negative so
\[ x - 2y + 4 \geq 0 \implies -2y \geq -x - 4 \implies y \leq \frac{1}{2}x + 2 \]

Therefore,
\[ D = \{(x, y) : y \leq \left(\frac{1}{2}\right)x + 2\} = \{(x, y) : x - 2y + 4 \geq 0\} \]

12.2.28: Graph several level curves of the function using the given window.
\[ z = 2x - y; \quad [-2, 2] \times [-2, 2] \]

This is a simple planar function so the level curves will be lines in the \( xy \)-plane. The level curves will have the form \( y = 2x - z_0 \) where \( z_0 \) is any fixed value.

**Soln:**

12.2.32: Graph several level curves of the function using the given window.
\[ z = \sqrt{y - x^2 - 1}; \quad [-5, 5] \times [-5, 5] \]

In this case the level curves will have the form \( y = x^2 + \left(z_0^2 + 1\right) \) where \( z_0 \) is any fixed value. Notice that these are parabolic level curves.

**Soln:**

12.2.34: Matching . . .

**Soln:**
* (a - B) the level curves of (a) are lines parallel to the \( x \)-axis
* (b - E) the level curves for (b) are hyperbolas
* (c - A) the plane in (c) is negative along the positive \( x \)-axis and along the negative \( y \)-axis
* (d - D) the level curves in (d) are circles
* (e - A) the plane in (e) is negative along the negative \( x \)-axis and along the positive \( y \)-axis.
* (f - F) the level curves of (f) are ellipses

12.2.59: The function has exactly one isolated peak or one isolated depression. Use a graphing utility to approximate the coordinates of the peak or depression.
\[ h(x, y) = 1 - e^{-(x^2 + y^2 - 2x)} \]
Figure 1: MATLAB plot

**Soln:** Using a graphing utility (like Wolfram Alpha or MATLAB) we get this picture. From the picture it appears that the minimum occurs at or near \((x, y) = (1, 0)\).

- Section 12.3: #22, #26, #42

12.3.22: Evaluate the limit

\[
\lim_{(x,y) \to (4,5)} \frac{\sqrt{x+y} - 3}{x+y-9}
\]

**Soln:** First observe that this function is not defined at the point \((4, 5)\) so we cannot simply plug in \(x = 4\) and \(y = 5\). Multiply by the algebraic conjugate of the numerator to get

\[
\lim_{(x,y) \to (4,5)} \frac{\sqrt{x+y} - 3}{x+y-9} = \lim_{(x,y) \to (4,5)} \left( \frac{\sqrt{x+y} - 3}{x+y-9} \right) \left( \frac{\sqrt{x+y} + 3}{\sqrt{x+y} + 3} \right)
\]

\[
= \lim_{(x,y) \to (4,5)} \frac{1}{(x+y-9)(\sqrt{x+y} + 3)}
\]

\[
= \frac{1}{\sqrt{4+5} + 3} = \frac{1}{6}
\]

12.3.26: Use the two-path test to prove that the limit does not exist

\[
\lim_{(x,y) \to (0,0)} \frac{4xy}{3x^2 + y^2}
\]

**Soln:** Along the line \(y = x\) we have

\[
\lim_{(x,y) \to (0,0)} \frac{4xy}{3x^2 + y^2} = \frac{4x^2}{4x^2} = 1.
\]

Along the line \(y = -x\) we have

\[
\lim_{(x,y) \to (0,0)} \frac{4xy}{3x^2 + y^2} = \frac{-4x^2}{4x^2} = -1.
\]

Since these limits do not agree the limit must not exist.

12.3.42: At what points of \(\mathbb{R}^2\) is the function continuous?

\[h(x, y) = \frac{\sqrt{x-y}}{4}\]

**Soln:** The square root function is continuous for all points in its domain so we simply need to find the domain of \(h\).

\[D(h) = \{(x, y) : x - y \geq 0\} = \{(x, y) : x \geq y\}\]